Lecture (30) Graphing polar Coordinates & Arc length

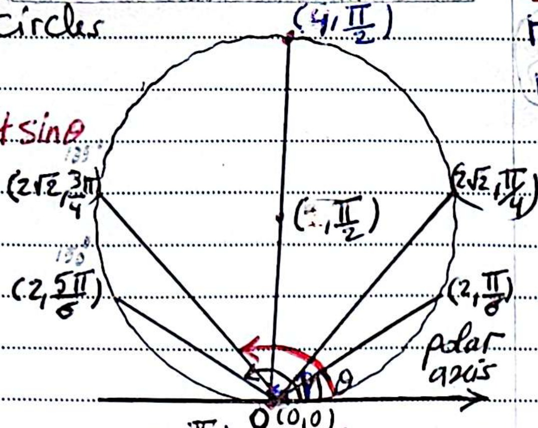
* The graph of Some polar Equations

[1] $r = a\cos\theta$ or $r = a\sin\theta$

are circles

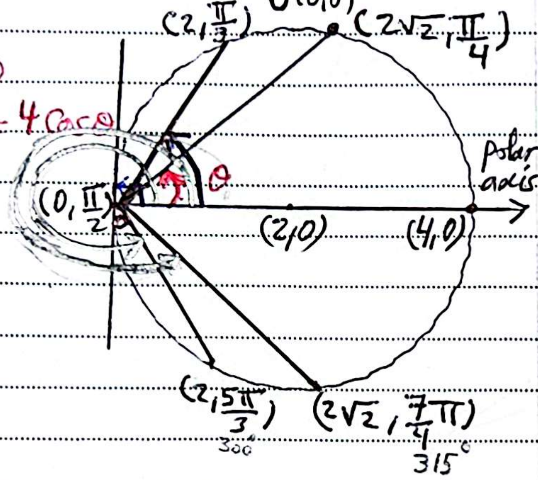
EX. ①

$r = 4\sin\theta$



EX. ②

$r = 4\cos\theta$

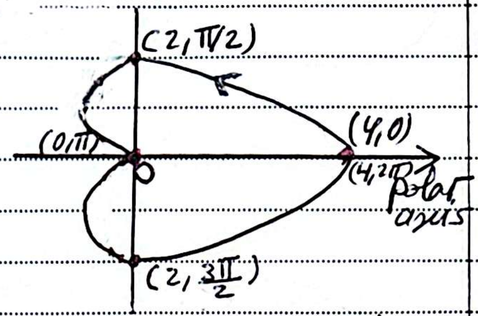


[2] Cardioid منحنى الكارديويد (المنحنى القلبي)

$r = a(1 \pm \cos\theta)$, $r = a(1 \pm \sin\theta)$ } are Cardioids

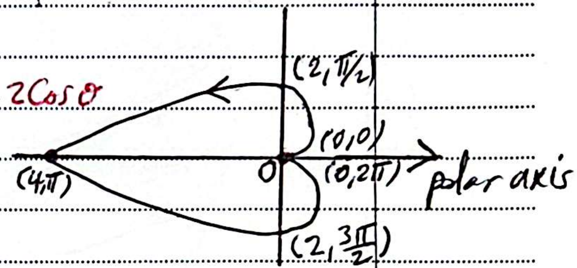
EX. ①

$r = 2 + 2\cos\theta$



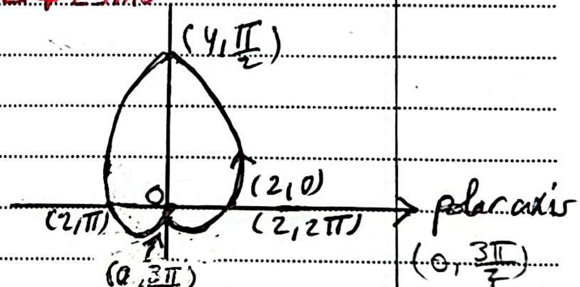
EX. ②

$r = 2 - 2\cos\theta$



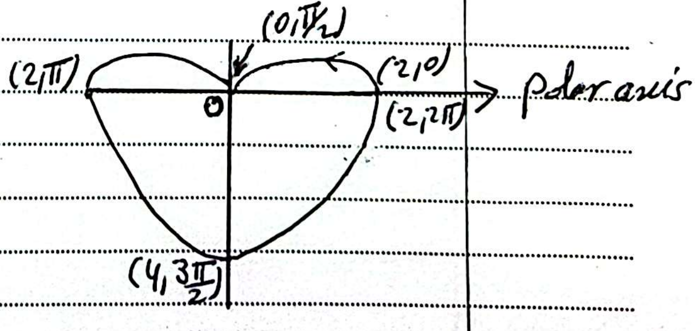
EX. ③

$r = 2 + 2\sin\theta$



EX. ④

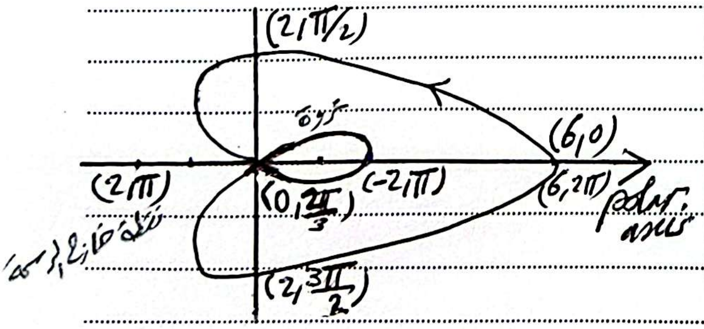
$r = 2 - 2\sin\theta$



[3] limacon منحنى الحلزون

$r = a \pm b\cos\theta$, $r = a \pm b\sin\theta$ } are limacons

EX: $r = 2 + 4\cos\theta$

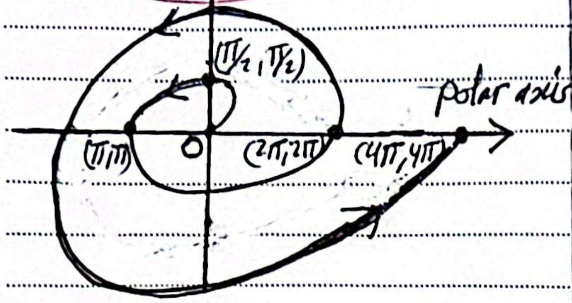


4) Spiral of archimedes

منحنى حلزون أرشميدس

$r = a\theta$

EX: $r = \theta$

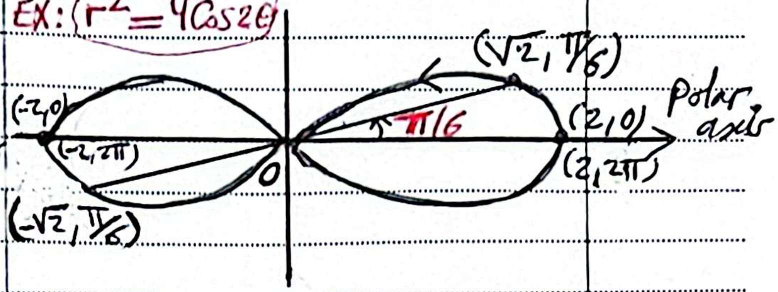


5) Lemniscate

منحنى الليمنسكيت

$r^n = a^n \cos n\theta$

EX: $r^2 = 4\cos 2\theta$

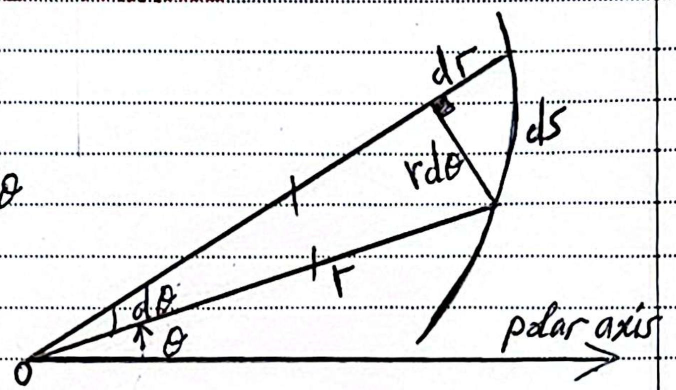


* Arc length in polar coordinates

$\therefore ds = \sqrt{(r\,d\theta)^2 + (dr)^2}$

i.e. $ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\; d\theta$

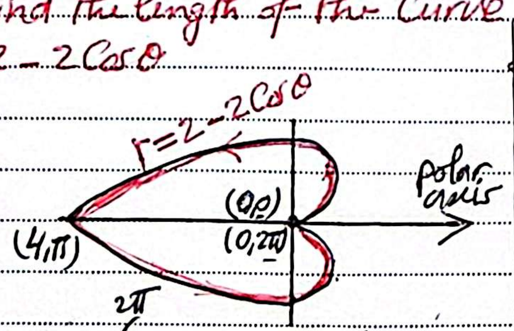
$\therefore$ Arc length of a curve by using polar coordinates is given by



$$\text{Arc length} = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\; d\theta$$

EX. Find the length of the curve $r = 2 - 2\cos\theta$

Ans:



$$\text{Arc length} = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\; d\theta$$

$$= \int_0^{2\pi} \sqrt{(2 - 2\cos\theta)^2 + (2\sin\theta)^2}\; d\theta$$

$$= \int_0^{2\pi} \sqrt{4 - 8\cos\theta + 4\cos^2\theta + 4\sin^2\theta}\; d\theta$$

$$\therefore \text{Arc length} = \int_0^{2\pi} \sqrt{8 - 8\cos\theta}\; d\theta$$

$$= 2\sqrt{2} \int_0^{2\pi} \sqrt{1 - \cos\theta}\; d\theta$$

$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

$$= 2\sqrt{2} \int_0^{2\pi} \sqrt{2}\,\sin\left(\frac{\theta}{2}\right) d\theta$$

$$= 8 \int_0^{2\pi} \sin\left(\frac{\theta}{2}\right) \cdot \frac{1}{2}\, d\theta$$

$$= 8\left[-\cos\left(\frac{\theta}{2}\right)\right]_0^{2\pi}$$

$= 8\,(1 + 1) = 16$ unit length.