Lecture ③ Change of Variables in Indefinite Integral "Method of substitution"

1

Theorem If F is an antiderivative of f then

$$\int f(g(x))\,g'(x)\,dx = F(g(x)) + C$$

and if $u = g(x) \Rightarrow du = g'(x)\,dx$

then $\int f(u)\,du = F(u) + C$

• **Ex** Evaluate the following Integrals.

① $\int \cos 4x\,dx$

Ans.

$I = \int \cos 4x\,dx$

$I = \frac{1}{4}\int \cos 4x \cdot 4dx$

$\therefore I = \frac{1}{4}\sin 4x + C$

OR let $u = 4x \Rightarrow du = 4dx$ i.e. $dx = \frac{du}{4}$

$I = \frac{1}{4}\int \cos u\,du$

$I = \frac{1}{4}\sin u + C$

$\therefore I = \frac{1}{4}\sin 4x + C$

② $\int x(2x^2+3)^{10}dx$

$I = \frac{1}{4}\int (2x^2+3)^{10}\,4x\,dx$

$I = \frac{1}{4}\,\frac{(2x^2+3)^{11}}{11} + C$

$I = \frac{(2x^2+3)^{11}}{44} + C$

OR, You can use $u = 2x^2+3$ ...

③ $\int x^2\sqrt[3]{3x^3+7}\,dx$

let $u = 3x^3+7$

$\Rightarrow du = 9x^2dx$

$\Rightarrow x^2dx = \frac{du}{9}$

$I = \frac{1}{9}\int \sqrt[3]{u}\,du$

$I = \frac{1}{9}\int u^{1/3}du$

$I = \frac{1}{9}\left(\frac{u^{4/3}}{4/3}\right) + C$

$I = \frac{1}{12}u^{4/3} + C$

$\therefore I = \frac{1}{12}(3x^3+7)^{4/3} + C$

④ $\int \sqrt{3x-2}\,dx$

$I = \frac{1}{3}\int \sqrt{3x-2}\;3dx$

$I = \frac{1}{3}\,\frac{(3x-2)^{3/2}}{3/2} + C$

$\therefore I = \frac{2}{9}(3x-2)^{3/2} + C$

OR, You can use $u = 3x-2$, ...

⑤ $\int \cos(4x-3)\,dx$

let $u = 4x-3 \Rightarrow du = 4\,dx$
$\Rightarrow dx = \frac{du}{4}$

$I = \frac{1}{4}\int \cos u\,du$

$I = \frac{1}{4}\sin u + C$

$\therefore I = \frac{1}{4}\sin(4x-3) + C$

⑥ $\int v \sin(v^2)\,dv$

$I = \frac{1}{2}\int \sin(v^2)\,2v\,dv$

$\therefore I = \frac{-1}{2}\cos(v^2) + C$

or, use $u = v^2$, ...

⑦ $\int \frac{\sin x}{\cos^4 x}\,dx$

$I = -\int \cos^{-4}x\,(-\sin x)\,dx$

$I = -\frac{\cos^{-3}x}{-3} + C$

$\therefore I = \frac{1}{3\cos^3 x} + C$

OR let $u = \cos x \Rightarrow du = -\sin x\,dx$
$\Rightarrow \sin x\,dx = -du$

$I = -\int u^{-4}\,du$

$I = -\left(\frac{u^{-3}}{-3}\right) + C$

$I = \frac{1}{3\cos^3 x} + C$

H.W

Evaluate each of the following Integral

pb① $\int (x^2+1)^9\,x\,dx$

Ans: $I = \frac{1}{20}(x^2+1)^{10} + C$

pb② $\int \frac{x^3+x}{\sqrt[3]{x^4+2x^2+3}}\,dx$

Ans: $I = \frac{3}{8}(x^4+2x^2+3)^{2/3} + C$

pb③ $\int \sqrt{x^2+2x}\,(x+1)\,dx$

Ans: $I = \frac{1}{3}(x^2+2x)^{3/2} + C$

*pb④ $\int (\sin x + \cos x)^2\,dx$

Ans: $I = x - \frac{1}{2}\cos 2x + C$

pb⑤ $\int x\sec^2(x^2+2)\,dx$

Ans: $I = \frac{1}{2}\tan(x^2+2) + C$

pb⑥ $\int \sec(5x-2)\tan(5x-2)\,dx$

Ans: $I = \frac{1}{5}\sec(5x-2) + C$

*pb⑦ $\int \sin x\,(1+\cos x)^2\,dx$

Ans: $I = -\frac{(1+\cos x)^3}{3} + C$

i.e. $I = -\cos x - \cos^2 x - \frac{1}{3}\cos^3 x + C$

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