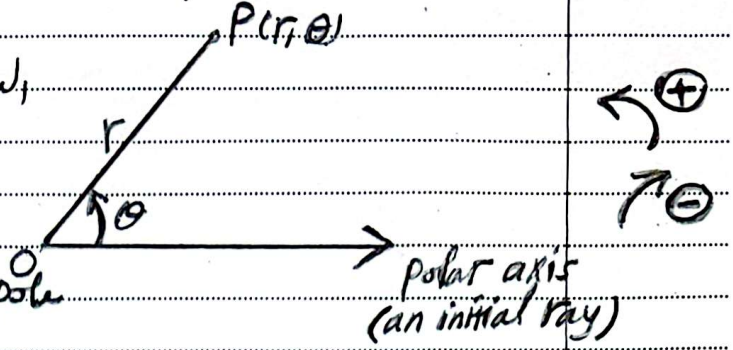
Lecture (29) Polar Coordinates

A point P is determined by r and θ in polar coordinates.

r is the polar distance (البعد القطبي)

θ is the measure of the angle formed by OP and the polar axis.



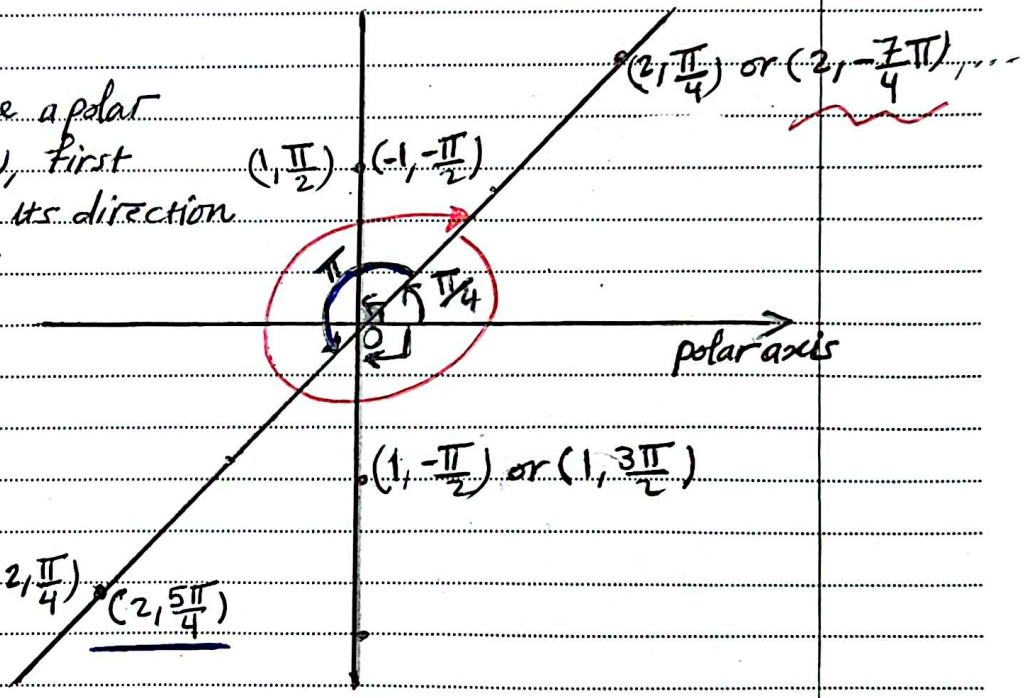
θ is positive when measured counterclockwise (عكس عقارب الساعة) direction and negative when measured clockwise.

Ex Plot each of the following polar coordinates

(a) $(1, \frac{\pi}{2})$ (b) $(-2, \frac{\pi}{4})$ (c) $(-1, -\frac{\pi}{2})$ (d) $(2, \frac{5\pi}{4})$
(e) $(2, -\frac{7}{4}\pi)$

Ans:

Note that to determine a polar coordinate pair (r, θ), first determine θ through its direction and then allocate r.



Notes

* For $r=2$, the complete list of angles is $\frac{\pi}{4}, \frac{\pi}{4} \pm 2\pi, \frac{\pi}{4} \pm 4\pi, \ldots$
i.e. the corresponding coordinate pairs of P are
$(2, \frac{\pi}{4} + 2n\pi)$, $n = 0, \pm 1, \pm 2, \ldots$
* A point in the plane has only one pair of Cartesian coordinates, but it has infinitely many pairs of polar coordinates.

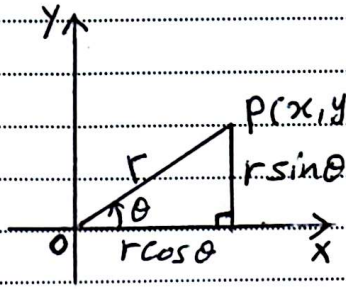
* The relation between polar and Cartesian coordinates in a plane.

$x = r\cos\theta$, $y = r\sin\theta$

$\Rightarrow r^2 = x^2 + y^2$,

$r = \sqrt{x^2+y^2}$,

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$



Ex(2) Find a polar equation for the circle $x^2+(y-3)^2=9$

Ans: $\because x^2+(y-3)^2=9$

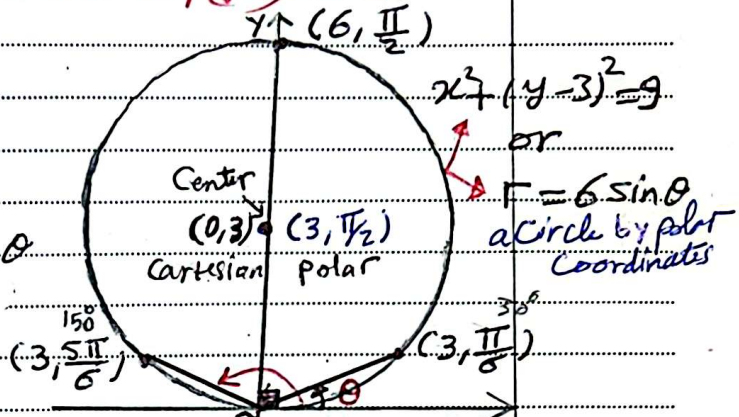
$x^2+y^2-6y+9=9$

$x^2+y^2-6y=0$,

$\because x^2+y^2=r^2$, $y=r\sin\theta$

$\therefore r^2-6r\sin\theta=0$

$\therefore r=6\sin\theta$



Ex(3) Replac the following polar equations by equivalent Cartesian equations and identify their graphs.

(a) $r\cos\theta = -4$ (b) $r^2 = 4r\cos\theta$ (c) $r = \frac{4}{2\cos\theta - \sin\theta}$

Ans: (a) $r\cos\theta = -4$

$\therefore x = -4$

The graph: Vertical line through $x=-4$

(b) $r^2 = 4r\cos\theta$

$x^2+y^2 = 4x$

$x^2-4x+y^2=0$

$x^2-4x+4+y^2=4$ by completing square

$(x-2)^2+y^2=4$

the graph: Circle with radius 2 and Center (2,0).

(c) $r = \frac{4}{2\cos\theta-\sin\theta}$

$r(2\cos\theta-\sin\theta)=4$

$2r\cos\theta - r\sin\theta = 4$

$2x-y=4$

$y=2x-4$

The graph: line with slope $m=2$ and y-intercept $b=-4$.

The polar Eqns $r=a\cos\theta$ and $r=a\sin\theta$ are circles