# Lecture (27) (III) Volume using Cylindrical shells (Slicing with Cylinders)

## The Shell Method

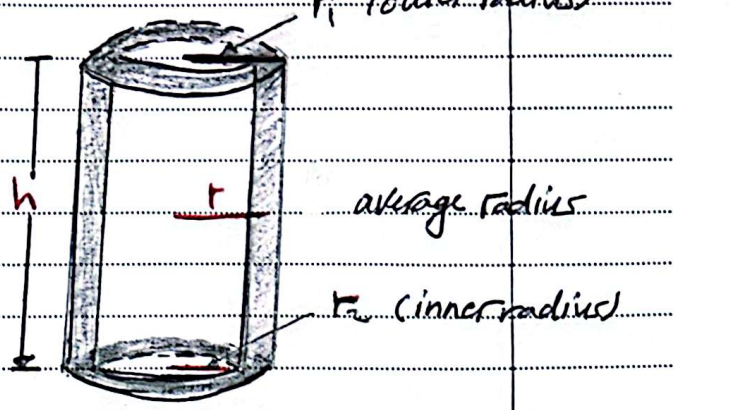
Volume of the Cylindrical shell (hollow circular cylinder) is

$$V = \pi r_1^2 h - \pi r_2^2 h$$

$$V = 2\pi \left(\frac{r_1 + r_2}{2}\right)(r_1 - r_2)h$$

$$\therefore V = 2\pi r h \Delta r$$

$r$: average radius, $h$: shell height, $\Delta r$: thickness




* Shell Formula for Revolution about the y-axis is

$$V = \int_a^b 2\pi \,(\text{shell radius})\,(\text{shell height})\,dx$$

i.e. $$V = \int_a^b 2\pi x f(x)\,dx$$

* Shell Formula for Revolution about the x-axis is

$$V = \int_c^d 2\pi \,(\text{shell radius})\,(\text{shell height})\,dy$$

i.e. $$V = \int_c^d 2\pi y g(y)\,dy$$

**Ex ①** The region bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$ is revolved about the y-axis to generate a solid. Find the volume of the solid.

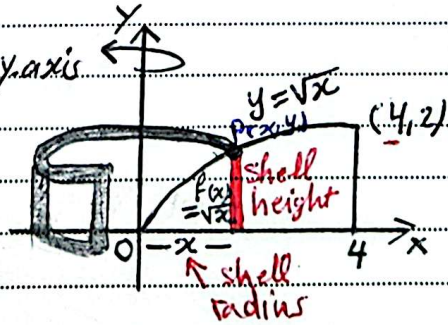
**Ans:**

The volume of the solid by revolving about y-axis is

$$V = \int_0^4 2\pi x f(x)\,dx$$

$$V = 2\pi \int_0^4 x\sqrt{x}\,dx = 2\pi \int_0^4 x^{\frac{3}{2}}\,dx$$

$$\therefore V = 2\pi \left[\frac{x^{5/2}}{5/2}\right]_0^4 = \frac{4\pi}{5}(2^5) = \frac{128}{5}\pi$$



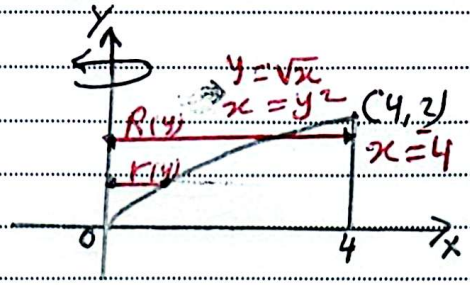
We can solve the previous Example by using Washer Method as follows:

$$V = \pi \int_0^2 ([R(y)]^2 - [r(y)]^2)\,dy$$

$$= \pi \int_0^2 [(4)^2 - (y^2)^2]\,dy$$

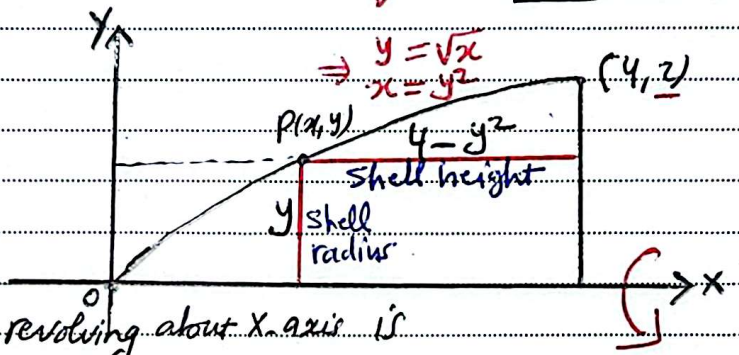
$$= \pi \int_0^2 (16 - y^4)\,dy =$$

$$= \pi \left[16y - \frac{y^5}{5}\right]_0^2 = 32\pi\left(1 - \frac{1}{5}\right) = \frac{128}{5}\pi.$$




EX② The region bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$ is revolved about x-axis to generate a solid. Find the volume of the solid by the shell method.

Ans:



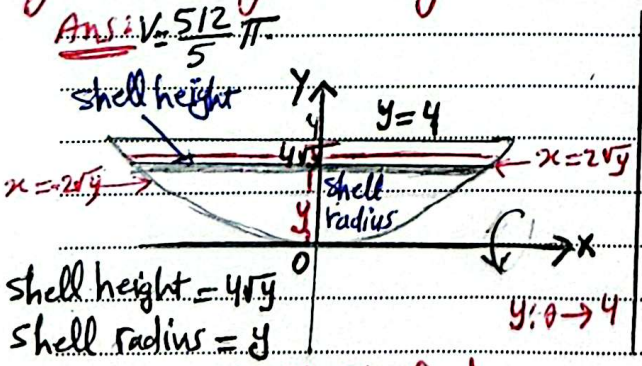

The volume of solid by revolving about x-axis is

$$V = \int_0^2 2\pi y(4 - y^2)\,dy = 2\pi \int_0^2 (4y - y^3)\,dy$$

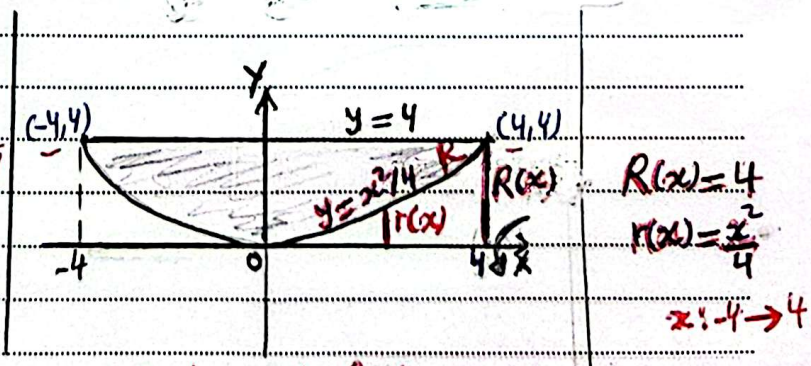
$$= 2\pi \left[2y^2 - \frac{y^4}{4}\right]_0^2 = 8\pi$$ which is the same result that was obtained before by using Disk Method, see Ex. ① Lecture (25)

HW Ex ③ Sketch the Region R bounded by the graphs of the equations $x^2 = 4y$, $y = 4$. Find the volume of the solid generated by revolving R about x-axis.

Ans: $V = \frac{512}{5}\pi$.




Shell Method




Washer Method