

Lecture (2)

Some Basic Rules For Indefinite Integral

Derivative

$$(1) \frac{d}{dx}(x) = 1$$

$$(2) \frac{d}{dx}\left(\frac{x^{r+1}}{r+1}\right) = x^r, r \neq -1$$

$$(3) \frac{d}{dx}(\sin x) = \cos x$$

$$(4) \frac{d}{dx}(\cos x) = -\sin x$$

$$(5) \frac{d}{dx}(\tan x) = \sec^2 x$$

$$(6) \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$(7) \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$(8) \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Indefinite Integral

$$\int dx = x + C, C \text{ is a constant.}$$

$$\int x^r = \frac{x^{r+1}}{r+1} + C \quad (\text{Power Rule})$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

Theorem (1)

$$(i) \int \frac{d}{dx}[f(x)] \, dx = f(x) + C$$

$$(ii) \frac{d}{dx} \int f(x) \, dx = f(x)$$

$$\frac{d}{dx} [F(x) + C] = F'(x) + 0 = f(x)$$

Theorem (2)

$$(i) \int c f(x) \, dx = c \int f(x) \, dx$$

$$(ii) \int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$



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Ex Evaluate each of the following Integral

(1) $\int (4x+3) dx$

Ans: $I = \frac{4x^2}{2} + 3x + C$

$I = 2x^2 + 3x + C$ #

(2) $\int (3\sqrt{u} + \frac{1}{\sqrt{u}}) du$

Ans: $I = 3(\frac{u^{3/2}}{3/2}) + \frac{u^{-1/2+1}}{1/2} + C$

$I = 2u^{3/2} + 2u^{1/2} + C$

$I = 2u^{3/2} + 2\sqrt{u} + C$ #

(3) $\int \sin(x+9) dx$

Ans: $I = -\cos(x+9) + C$

(4) $\int \sin 9x dx$

Ans: $I = \frac{1}{9} \int \sin 9x \cdot 9 dx$

$I = \frac{-1}{9} \cos 9x + C$ #

• OR you can use the substitution

$u = 9x \Rightarrow dx = \frac{du}{9}$

$\therefore I = \int \sin u \frac{du}{9}$

$I = \frac{1}{9} \int \sin u du$

$I = \frac{-1}{9} \cos u + C$

$I = \frac{-1}{9} \cos 9x + C$ #

(5) $\int \frac{(1 + \cot^2 z) \cot z}{\csc z} dz$

Ans: $I = \int \frac{\csc^2 z \cot z}{\csc z} dz$

$I = \int \csc z \cot z dz$

$I = -\csc z + C$ #

(6) $\int \frac{d}{dx} (\sin \sqrt[3]{x}) dx$

Ans: $I = \sin \sqrt[3]{x} + C$ #

(7) $\frac{d}{dx} \int \sin \sqrt[3]{x} dx$

Ans: $I = \sin \sqrt[3]{x}$ #

(8) $\int \csc u \cot u \sec u du$

Ans: $I = \int \csc u \frac{\cos u}{\sin u} \sec u du$

$I = \int \csc^2 u du$

$I = -\cot u + C$ #

(9) $\int \frac{\sec t}{\cos t} dt$

Ans: $I = \int \sec^2 t dt$

$I = \tan t + C$ #