Lecture (16) Trigonometric Integrals (Continued)

- Integrals of powers of tan x and sec x

To evaluate $\int \tan^m x \sec^n x\,dx$

m is an odd Integer	n is even Integer	m is even and n is odd
Use $\tan^2 x = \sec^2 x - 1$, $u = \sec x$, $du = \sec x \tan x\,dx$	Use $\sec^2 x = 1 + \tan^2 x$, $u = \tan x$, $du = \sec^2 x\,dx$	You can use Integration by parts

Ex. Evaluate the following Integrals

① $\int \tan^2 x \sec^4 x\,dx$

Use $\sec^2 x = 1 + \tan^2 x$,

$u = \tan x$, $du = \sec^2 x\,dx$

$I = \int \tan^2 x \underbrace{(1+\tan^2 x)}_{\sec^2 x} \sec^2 x\,dx$

$I = \int u^2(1+u^2)\,du$

$I = \int (u^2 + u^4)\,du$

$I = \frac{1}{3}u^3 + \frac{1}{5}u^5 + C$

$\therefore I = \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + C$

③ $\int \tan^4 x \sec^4 x\,dx$ H-W

Hint: Use $\sec^2 x = 1 + \tan^2 x$, ... as in ①.

Ans:

$I = \frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + C$

② $\int \tan^3 x \sec^5 x\,dx$

Use $\tan^2 x = \sec^2 x - 1$,

$u = \sec x$, $du = \sec x \tan x\,dx$

$I = \int \tan^2 x \tan x \sec^4 x \sec x\,dx$

$I = \int (\sec^2 x - 1)\sec^4 x \sec x \tan x\,dx$

$I = \int (\sec^6 x - \sec^4 x)\sec x \tan x\,dx$

$I = \int (u^6 - u^4)\,du$

$I = \frac{1}{7}u^7 - \frac{1}{5}u^5 + C$

$\therefore I = \frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C$

④ $\int \tan^2 x \sec^3 x\,dx$ H.W

Hint: Use $\tan^2 x = \sec^2 x - 1$ ... $\overset{\text{i.e.}}{\Rightarrow}$ $I = \int \sec^5 x\,dx - \int \sec^3 x\,dx$

Then use Integration by parts for both Integrals ....

... ...

Ans: $I = \frac{1}{4}\sec^3 x \tan x - \frac{1}{8}\sec x \tan x - \frac{1}{8}\ln|\sec x + \tan x| + C$.

Products of Sines and Cosines.

To evaluate the Integrals $\int \sin mx \cos nx \, dx$,

$\int \cos mx \cos nx \, dx$ and $\int \sin mx \sin nx \, dx$

Use the following identities.

(1) $\sin mx \cos nx = \frac{1}{2}\left[\sin(\overset{\text{الفرق}}{m-n})x + \sin(\overset{\text{المجموع}}{m+n})x\right]$

(2) $\cos mx \cos nx = \frac{1}{2}\left[\cos(m-n)x + \cos(m+n)x\right]$

(3) $\sin mx \sin nx = \frac{1}{2}\left[\cos(m-n)x - \cos(m+n)x\right]$

اختلاف

$\oplus \to \ominus$

EX Evaluate the following Integrals

① $\int \sin 3x \cos 2x \, dx$

$I = \frac{1}{2}\int (\sin x + \sin 5x)\,dx$

$\therefore I = \frac{-1}{2}\cos x - \frac{1}{10}\cos 5x + C$

② $\int \sin\theta \sin 3\theta \, d\theta$

$I = \frac{1}{2}\int [\cos 2\theta - \cos 4\theta]\,d\theta$, $\cos(-2\theta) = \cos 2\theta$

$I = \frac{1}{4}\sin 2\theta - \frac{1}{8}\sin 4\theta + C$

③ $\int \cos 5x \cos 2x \, dx$

$I = \frac{1}{2}\int [\cos 3x + \cos 7x]\,dx$

$\therefore I = \frac{1}{6}\sin 3x + \frac{1}{14}\sin 7x + C$

④ $\int \sin 3x \cos 5x \, dx$ HW

Ans:

$I = \frac{-\cos 8x}{16} + \frac{\cos 2x}{4} + C$

Note that $\sin(-2x) = -\sin 2x$