"Revision" (I)

Lecture (13)⁺ Some pbs for evaluating "Derivatives & Integrals"

Pb 1 Find $\frac{dy}{dx}$ if

(i) $y=\pi^{\sin x}$ (ii) $y=(\ln x)^{\tan x}$

Ans:

(i) $y=\pi^{\sin x}$

$$\frac{dy}{dx}=y'=\pi^{\sin x}\ln\pi\cos x$$
$$=\ln\pi\,\pi^{\sin x}\cos x$$

(ii) $y=(\ln x)^{\tan x}$

Take ln for both sides

$\ln y=\tan x\ln|\ln x|$

Diff both sides w.r.t x

$$\frac{1}{y}y'=\tan x\frac{1}{\ln x}\frac{1}{x}+\ln|\ln x|\sec^2 x$$

$$\therefore y'=(\ln x)^{\tan x}\left[\frac{\tan x}{x\ln x}+\sec^2 x\ln|\ln x|\right]$$

Pb 2 Evaluate $\int\frac{x+2}{x^2+4}dx$

Ans:

$$I=\int\frac{x+2}{x^2+4}dx$$

$$I=\int\frac{x}{x^2+4}dx+2\int\frac{1}{x^2+4}dx$$

$$I=\frac{1}{2}\int\frac{2x}{x^2+4}dx+2\int\frac{1}{x^2+4}dx$$

$$\therefore I=\frac{1}{2}\ln(x^2+4)+\tan^{-1}\left(\frac{x}{2}\right)+C$$

Pb 3 Evaluate $\int\frac{1}{x\sqrt{x^4-9}}dx$

Ans:

$$I=\int\frac{x}{x^2\sqrt{x^4-9}}dx \qquad \frac{\times x}{\times x}$$

$$I=\frac{1}{2}\int\frac{2x}{x^2\sqrt{x^4-9}}dx$$

$$\therefore I=\frac{1}{2}\left(\frac{1}{3}\right)\sec^{-1}\left(\frac{x^2}{3}\right)+C$$

$$I=\frac{1}{6}\sec^{-1}\left(\frac{x^2}{3}\right)+C$$

Pb 4 Evaluate $\int\frac{e^x}{\sqrt{1-e^{2x}}}dx$

$u=e^x\Rightarrow du=e^x dx$

$$I=\int\frac{e^x}{\sqrt{1-e^{2x}}}dx$$

$$I=\int\frac{1}{\sqrt{1-u^2}}du$$

$$I=\sin^{-1}u+C$$

$$\therefore I=\sin^{-1}(e^x)+C$$

Pb 5 Evaluate $\int\pi^{\sin x}\cos x\,dx$

$$I=\frac{\pi^{\sin x}}{\ln\pi}+C$$

Pb 6 Evaluate $\int_{-1}^{0}\frac{x}{x^2+5}dx$

$$I=\frac{1}{2}\int_{-1}^{0}\frac{2x}{x^2+5}dx$$

$$I=\frac{1}{2}\left[\ln(x^2+5)\right]_{-1}^{0}$$

$$I=\frac{1}{2}(\ln 5-\ln 6)$$

$$I=\frac{1}{2}\ln\left(\frac{5}{6}\right)=\ln\sqrt{\frac{5}{6}}$$

Pb 7 Evaluate $\int \frac{x^3}{x^2+1}\,dx$

Ans:

let $u = x^2+1 \Rightarrow du = 2x\,dx$,

$x^2 = u-1$

$$I = \int \frac{x^3}{x^2+1}\,dx = \frac{1}{2}\int \frac{x^2 \cdot 2x}{x^2+1}\,dx$$

$$I = \frac{1}{2}\int \left(\frac{u-1}{u}\right) du$$

$$I = \frac{1}{2}\int \left(1 - \frac{1}{u}\right) du$$

$$I = \frac{1}{2}\left(u - \ln|u|\right) + C$$

$$I = \frac{1}{2}\left(x^2 + 1 - \ln(x^2+1)\right) + C$$

$$\therefore I = \frac{1}{2}x^2 - \frac{1}{2}\ln(x^2+1) + C$$

Hint: You can use long division directly.

Pb 8 Evaluate $\int \frac{1}{x \ln \sqrt[3]{x}}\,dx$

Ans: $\ln \sqrt[3]{x} = \frac{1}{3}\ln x$

$$I = \int \frac{1}{x \ln \sqrt[3]{x}} = \frac{1}{3}\int \frac{1}{\ln x}\cdot\frac{1}{x}\,dx$$

$$= \frac{1}{3}\int \frac{1/x}{\ln x}\,dx$$

$$= \frac{1}{3}\ln|\ln x| + C$$

Pb 9 Evaluate $\int \frac{e^x}{\cosh x}\,dx$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$I = \int \frac{e^x}{\cosh x}\,dx = 2\int \frac{e^x}{e^x + e^{-x}}\,dx \qquad \frac{\times e^x}{\times e^x}$$

$$I = \int \frac{2e^{2x}}{e^{2x}+1}\,dx$$

$$\therefore I = \ln(e^{2x}+1) + C$$

Pb 10 Evaluate

$$\int \frac{x^2+1}{x\sqrt{4-x^2}}\,dx$$

Ans:

$$I = \int \frac{x^2+1}{x\sqrt{4-x^2}}\,dx$$

$$= \int \frac{x^{\cancel{2}}}{\cancel{x}\sqrt{4-x^2}}\,dx \qquad \frac{x-2}{x-2}$$

$$+ \int \frac{1}{x\sqrt{4-x^2}}\,dx$$

$$= \frac{-1}{2}\int \frac{-2x}{\sqrt{4-x^2}}\,dx \Rightarrow$$ البسط مشتقة ما تحت الجذر

$$+ \int \frac{1}{x\sqrt{4-x^2}}\,dx \Rightarrow \operatorname{sech}^{-1}$$

$$\therefore I = \frac{-1}{2}\left(2\sqrt{4-x^2}\right)$$

$$-\frac{1}{2}\operatorname{sech}^{-1}\left(\frac{|x|}{2}\right) + C$$

$$I = -\sqrt{4-x^2} - \frac{1}{2}\operatorname{sech}^{-1}\left(\frac{|x|}{2}\right) + C$$

Hint $\int \frac{1}{u\sqrt{a^2-u^2}}\,du = \frac{-1}{a}\operatorname{sech}^{-1}\left(\frac{|u|}{a}\right) + C$

$\int \frac{1}{\sqrt{u}}\,du = 2\sqrt{u} + C$ البسط مشتقة ما تحت الجذر 2X

& $\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C$, $|u| > a$

Pb 11 HW Evaluate

$$\int \frac{x^2+1}{x\sqrt{x^2-4}}\,dx$$

Ans:

$$I = \sqrt{x^2-4} + \frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right) + C, \quad |x| > 2$$