Chapter # 6:

The Free Electron Model

Lecture 1: The Free Electron Model: Classical Description of Electron Transport in Metals

6-1 Introduction

In our daily lives, we find that metals (such as iron, copper, gold, silver, …) play an important role.

Among the most important physical properties that characterize these metals are:

1. High mechanical strength
2. High density
3. Good electrical and thermal conductivity
4. High optical reflectivity

Why do we need to study electrons?

- Electrons are responsible for electrical conduction.
- Electrons determine optical properties.
- Electrons determine the magnetic moment.
- Electrons determine the superconductivity of materials.

- **Free electron gas = framework/assumption** (electrons move freely, weak lattice interaction)
- **Drude** = classical treatment (Newton + relaxation time)
- **Sommerfeld** = quantum treatment (Fermi–Dirac + Pauli principle)

6-2 Free Electron Gas

The free electron model, which is the basis of this chapter, assumes that conduction electrons are completely free. Thus, the total energy is purely kinetic, except for the potential energy at the metal surface, which confines the free electrons within the material, as shown in Figure 1.

This figure illustrates that electrons are free to move inside the metal where the potential energy is constant, but are confined by very high potential barriers at the surface. This simplification allows us to treat electrons as free particles within a box, while preventing them from leaving the material.

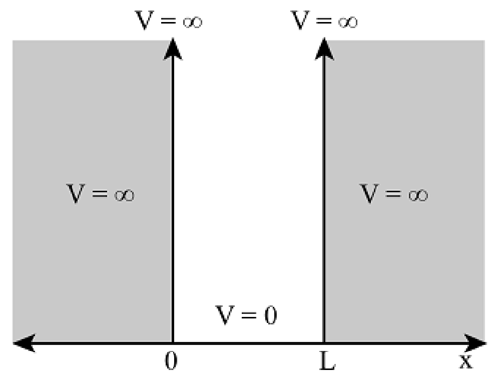

Figure (1): Potential energy in the free electron model.

This model assumes only instantaneous collisions between free electrons and ions. This situation is very similar to an ideal gas; therefore, this model is called the **"free electron gas."**

This approximation neglects the detailed interaction between electrons and the periodic lattice of ions.

This is a simplified quantum picture that we will use later; for now, we focus on the classical description.

The Drude model (1900) is a classical framework that applies kinetic gas theory to electrons in metals, treating them as a "free electron gas" that drifts under electric fields and scatters off immobile positive ion cores. It successfully explains Ohm's law, electrical/thermal conductivity, and the Hall effect, despite ignoring electron-electron interactions and quantum mechanics.

6-3 Electrical Conductivity

The conductivity law in metals (Ohm's law) is given by:

$$V = IR \qquad (1)$$

It can also be written as:

$$J = \sigma E \qquad (2)$$

where:

- J: current density = $\frac{I}{A}$

- σ: electrical conductivity
- E: electric field

Since resistivity ρ is the inverse of conductivity:

$$\rho = \frac{1}{\sigma} \quad \text{where,} \qquad \rho = \frac{R\,A}{l} \qquad (3)$$

The classical model derives electrical conductivity assuming that current results from the motion of conduction electrons under the influence of an electric field.

Thus, the electric field exerts a force of −eE on the electron. There is also a **drag force** due to collisions with the surrounding medium, equal to:

$$-\frac{m\mathrm{v}_d}{\tau} \qquad (4)$$

Applying Newton's law:

$$m\frac{d\mathrm{v}_d}{dt} = -eE - \frac{m\mathrm{v}_d}{\tau} \qquad (5)$$

where v_d is the drift velocity.

Electrons accelerate under E but collisions randomize motion → average velocity = drift velocity.

τ is the average time an electron moves freely before scattering.

In the **steady-state condition**, we get:

$$\vec{v}_d = -\frac{e\tau}{m}\vec{E} \qquad (6)$$

Since:

$$J = \frac{I}{A} = \frac{\Delta q}{\Delta t\, A} = \frac{\Delta q}{\Delta t}\frac{\Delta x}{V} = \frac{(-Ne)}{V}v_d = -ne\,v_d \qquad (7)$$

and using J=σE, we obtain:

$$J = -ne\left(-\frac{e\tau}{m}E\right) = \frac{ne^2\tau}{m}E = \sigma E \qquad (8)$$

Then, $$\sigma = \frac{ne^2\tau}{m} \qquad (9)$$

This means that electrical conductivity is proportional to the **relaxation time** (the time between successive collisions).

6-4 Heat Capacity

We use here the thermal conductivity model of gases.

The average kinetic energy of a free particle at temperature T is:

$$E = \frac{3}{2}kT \qquad (10)$$

Thus:

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$v = \sqrt{\frac{3kT}{m}} \qquad (11)$$

The average energy per mole is:

$$\langle E \rangle = \frac{3}{2}N_A kT \qquad (12)$$

Thus, the heat capacity is:

$$C = \frac{\partial E}{\partial T} = \frac{3}{2}R \qquad (13)$$

This represents the **electronic contribution** to heat capacity.

The total heat capacity includes contributions from electrons and phonons:

$$C = 3R + \frac{3}{2}R = \frac{9}{2}R = 4.5R \qquad (14)$$

However, this contradicts experimental results, which show that the heat capacity of metals at high temperatures equals:

$$C = 3R$$

This result disagrees with experiments, indicating that the classical model overestimates the contribution of electrons to heat capacity.

A more accurate description of how energy is distributed among electrons is therefore required.

6-5 Thermal Conductivity:

Thermal conductivity K is given by:

$$K = \frac{1}{3} C\, v\, l \qquad (15)$$

where l is the mean free path of electrons, while v represents the average thermal speed of electrons responsible for transporting heat inside the metal.

In classical theory, this speed is determined by temperature, but in reality, this speed is not purely determined by temperature, as will be discussed later.

Thermal conductivity is a measure of how easily heat is transferred through a material.

Substituting equations (11) and (13) into (15):

$$K = \frac{1}{3} \left(\frac{3}{2} R\right)\left(\sqrt{\frac{3kT}{m}}\right) l \qquad (16)$$

Since: $l = \tau$ v

We obtain:

$$K = \frac{3RkT}{2m} \tau \qquad (17)$$

Thermal conductivity increases with relaxation time, meaning fewer collisions allow better heat transport.

At this stage, we introduce electrical conductivity, heat capacity, and thermal conductivity because they are directly measurable properties that depend on electron behavior in metals. In this lecture, we present their classical description. Later, by analyzing these properties within the free electron model, we can test how well the model describes reality and identify its limitations.