Chapter #4:
Vibrations in a Crystal

Lecture 6: Phonon

Introduction

https://www.youtube.com/shorts/AHtehJ0DQWY

The vibrational energy of a crystal lattice is **quantized** in a way similar to the quantization of electromagnetic radiation.

Classical vs Quantum Description

	Classical Mechanics	Quantum mechanics
Light	Electromagnetic wave $\vec{E} = E_0 e^{i(\vec{q}.\vec{r}-\omega t)}\hat{r}$	Light has energy $\hbar\omega$. The virtual particle of light is called a "photon".
Crystal vibration	Back-and-forth wave $\vec{u}_q(\vec{R}_l^0) = u_0 e^{i(\vec{q}.\vec{R}_l^0-\omega t)}\hat{r}$	Vibration in a crystal has energy $\hbar\omega$. The virtual particle of vibration is called "phonon".

Definition of Phonon

Therefore, the **quantized unit of energy** associated with a vibrational (sound) wave inside a crystal is called a:

👉 **Phonon**

Phonon Energy

The energy of a phonon is given by:

$$E_n = (n + \frac{1}{2})\hbar\omega \qquad (1)$$

Where:

- n: represents an integer quantum number

◇ Ground State Energy

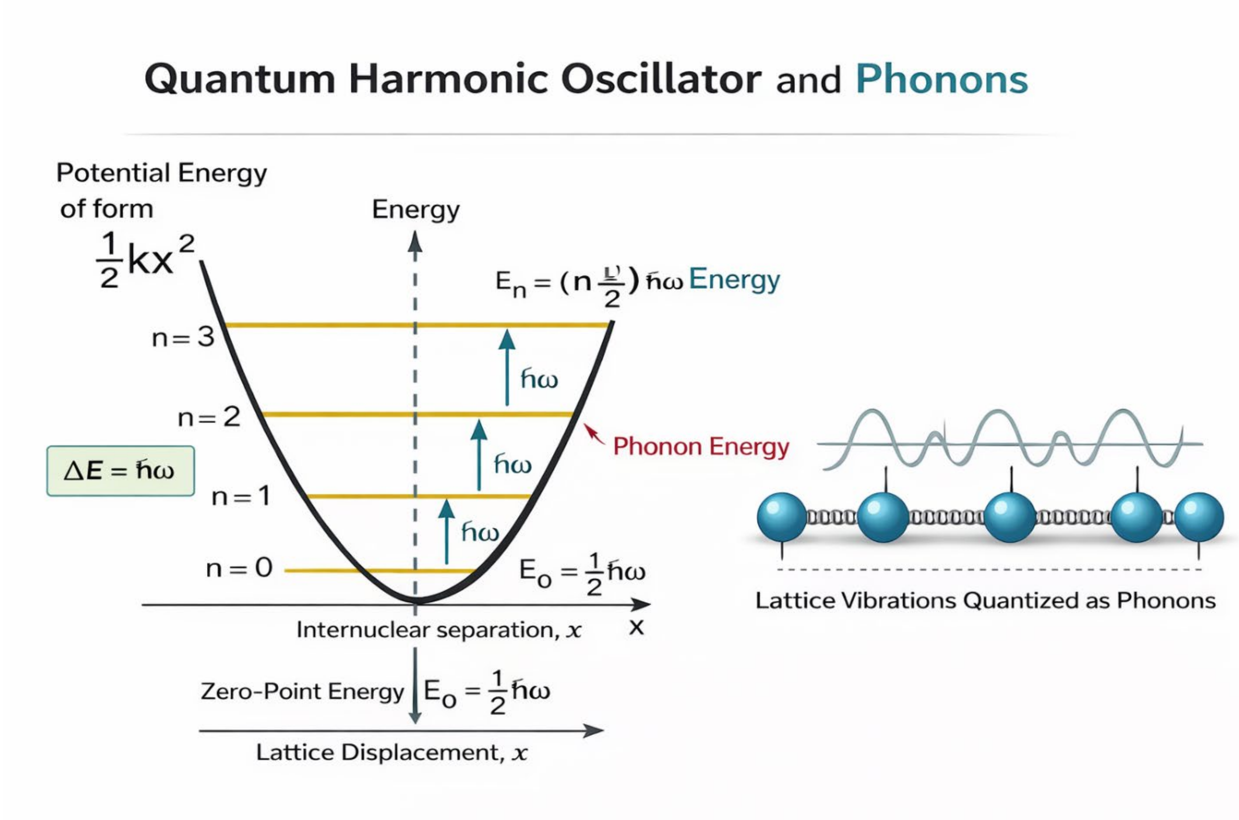
When: n=0

$$E_0 = \frac{1}{2}\hbar\omega$$

👉 This means:

The oscillator still has motion even in the ground state (n=0)
(This is called *zero-point energy*)

- Energy in electromagnetic waves is carried in quanta called **photons**.
- Energy associated with vibrations of a crystal lattice is carried in quanta called **phonons**.



📊 In this figure:

- ➢ Each horizontal line → energy level
- ➢ Distance between lines → $\hbar\omega$
- ➢ Arrow → creation/annihilation of phonon (similar to photon emission/absorption in atoms).

📊 As shown in *the figure*, the harmonic oscillator has discrete energy levels:

- n=0,1,2,3,…
- Each level is separated by $\hbar\omega$
- Each normal mode of lattice vibration behaves like a quantum harmonic oscillator.

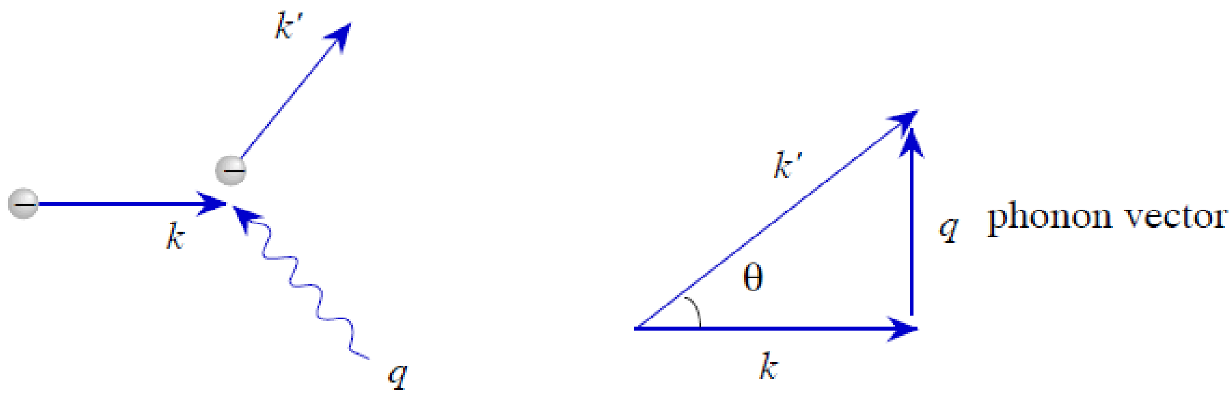
- A phonon corresponds to the transition between two adjacent energy levels.
- ω here is the same frequency from the dispersion relation ω(q) discussed in the last three lectures.

Phonon Interactions

Phonons can interact with other particles, such as:

- Photons
- Neutrons
- Electrons

Elastic Scattering



In the case of **elastic scattering** of an X-ray photon inside a crystal:

$$\vec{K}` = \vec{K} + \vec{G} \qquad (2)$$

Where:

- $\vec{K}$: incident photon wave vector
- $\vec{K}`$: scattered photon wave vector
- $\vec{G}$: reciprocal lattice vector

👉 This is the **diffraction condition** (Bragg condition in reciprocal space)

▧ Inelastic Scattering

When scattering is **inelastic**, the selection rule becomes:

$\vec{K}` \pm q = \vec{K} + \vec{G}$ (3)

More precisely,

When a phonon is involved:

- Energy changes:

$$E_f = E_i \pm \hbar\omega$$

- Momentum (more precisely, crystal momentum) changes:

$$k_f = k_i \pm q$$

◇ Where:

- $q^{\rightarrow}$: phonon wave vector

◇ Interpretation of Signs:

- (+): phonon is **created**
- (−): phonon is **annihilated**

📊 As illustrated in *the figures*:

- Momentum conservation involves:
  - photon
  - phonon
  - reciprocal lattice

▧ Physical Meaning

- In elastic scattering → no energy exchange
- In inelastic scattering → energy is exchanged with the lattice.

➢ Elastic → no phonon
➢ Inelastic → phonon created or destroyed

👉 This is how phonons are **experimentally observed.**

Option:

You can also find more discussion on phonon and dispersion relation by watching this lecture:

https://www.youtube.com/watch?v=UnHqQw834Z0