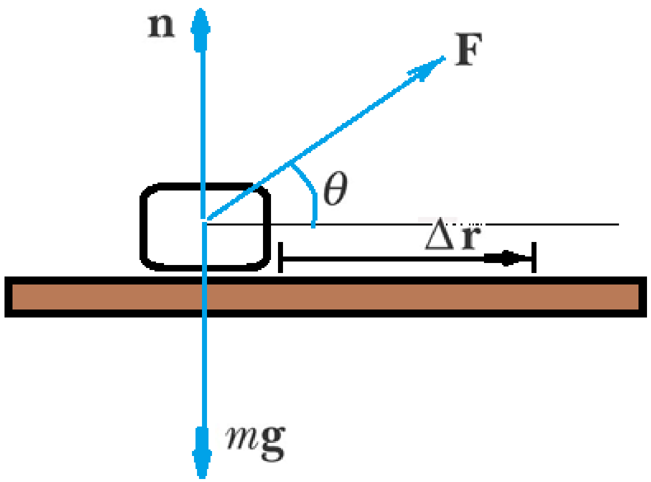
Work and Kinetic Energy

Chapter 7

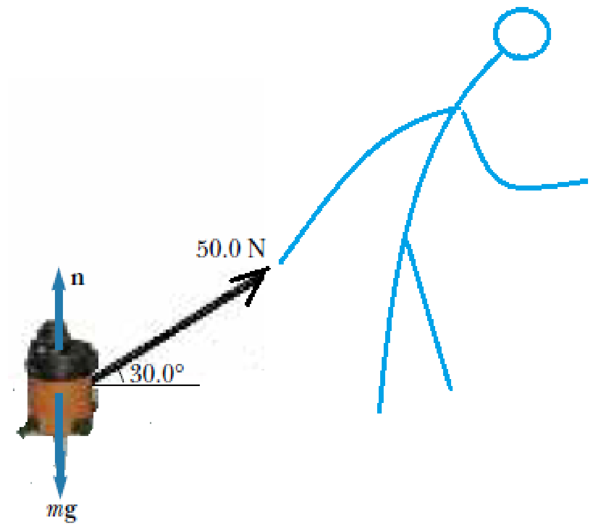
7.1 WORK DONE BY A CONSTANT FORCE

- The work, *W*, done on a system by an agent exerting a constant force on the system is the product of the magnitude, *F*, of the force, the magnitude Δr of the displacement of the point of application of the force, and cos θ, where θ is the angle between the force and the displacement vectors
  - $W = F \Delta r \cos \theta$
- The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application



- Work is a scalar quantity
- The unit of work is a joule (J)
  - 1 joule = 1 newton · 1 meter
  - J = N · m
- ***Example 7.1:***

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50$ N at an angle of 30.0° with the horizontal Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

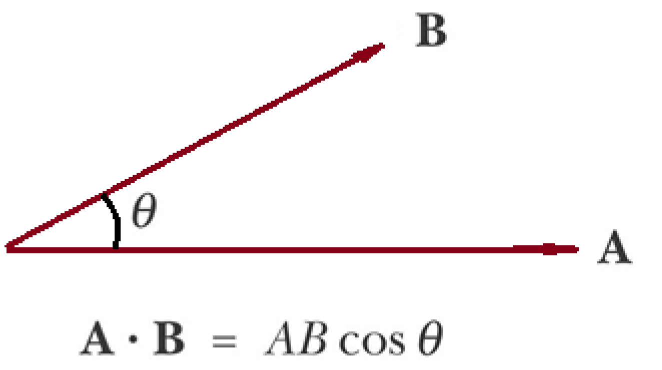


- ***Solution:***

$$W = (F\cos\theta)\,d$$
$$= (50.0\text{ N})(\cos 30.0°)(3.00\text{ m}) = 130\text{ N}\cdot\text{m}$$
$$= \boxed{130\text{ J}}$$

7.2 THE SCALAR PRODUCT OF TWO VECTORS

- The scalar product of two vectors is written as $\mathbf{A} \cdot \mathbf{B}$
  - It is also called the dot product
    - $\mathbf{A} \cdot \mathbf{B} = A\ B \cos\ \theta$
  - θ is the angle *between* *A* and *B*



- Dot Products of Unit Vectors

$$\hat{\text{i}} \cdot \hat{\text{i}} = \hat{\text{j}} \cdot \hat{\text{j}} = \hat{\text{k}} \cdot \hat{\text{k}} = 1$$

$$\hat{\text{i}} \cdot \hat{\text{j}} = \hat{\text{i}} \cdot \hat{\text{k}} = \hat{\text{j}} \cdot \hat{\text{k}} = 0$$

- Using component form with **A** and **B**:

$$A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

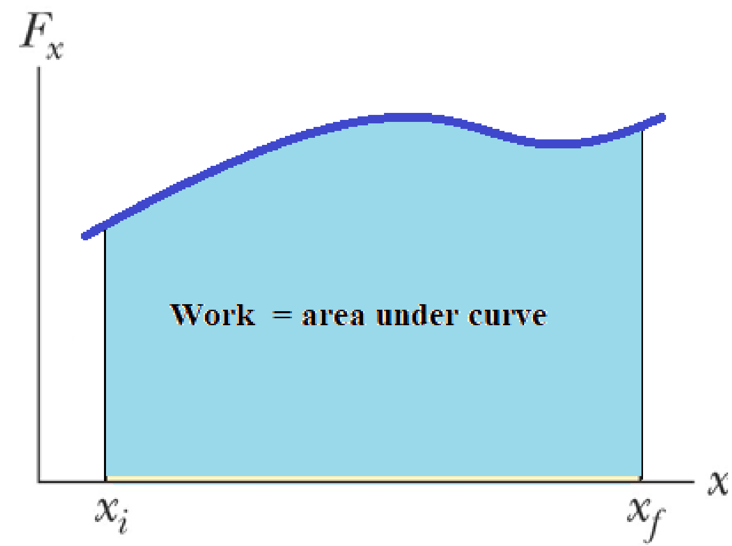
$$B = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$A \cdot B = A_xB_x + A_yB_y + A_zB_z$$

- Work Done by a Varying Force:

$$W = \int_{x_i}^{x_f} F_x dx$$

- The work done is equal to the area under the curve



- If more than one force acts on a particle, the total work done on the system is the work done by the net force

$$\sum W = W_{net} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx$$

- ***Example 7.2 :***

The vectors A and B are given by

A = 2i + 3j and B = - i + 2j.

(a) Determine the scalar product (A.B).

(b) Find the angle between A and B.

- ***Solution:***

- (a) $$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= (2\mathbf{i} + 3\mathbf{j})\cdot(-\mathbf{i} + 2\mathbf{j}) \\ &= -2\mathbf{i}\cdot\mathbf{i} + 2\mathbf{i}\cdot 2\mathbf{j} - 3\mathbf{j}\cdot\mathbf{i} + 3\mathbf{j}\cdot 2\mathbf{j} \\ &= -2(1) + 4(0) - 3(0) + 6(1) \\ &= -2 + 6 = \boxed{4}\end{aligned}$$

- (b)

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{8.06} = \boxed{60.2^\circ}$$

7.3 WORK DONE BY A VARYING FORCE

- **Work Done by a Spring:**
- The force exerted by the spring is

  $F_s = - kx$

  - x is the position of the block with respect to the equilibrium position ($x = 0$)
  - k is called the spring constant or force constant and measures the stiffness of the spring
- This is called Hooke's Law
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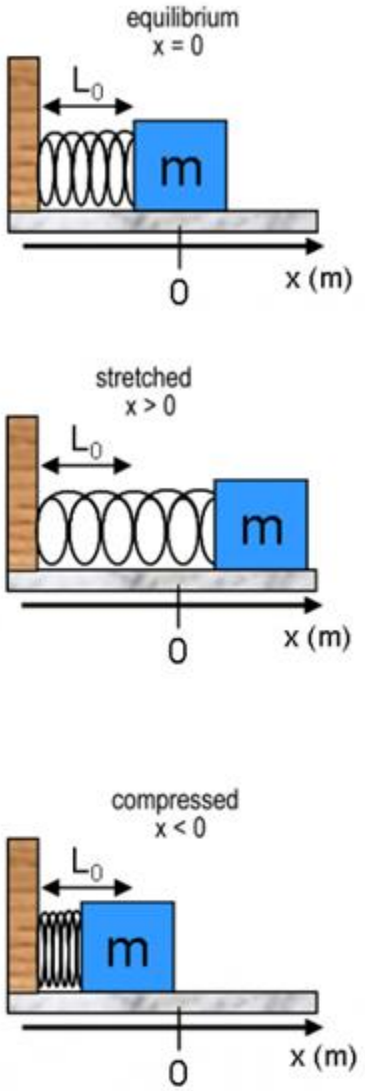
- Work Done by a Spring

$$W_s = \int_{x_i}^{x_f} F_x dx = \frac{1}{2} k x_{max}^2$$



- ***Example 7.3 :***
- Determining the force constant *k* of a spring. The elongation *d* is caused by the attached object, which has a weight *mg*.
- ***Solution:***

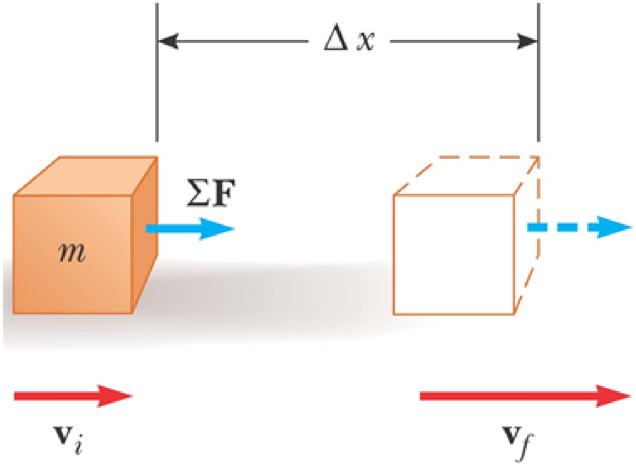
$$|\mathbf{F}_s| = kd = mg,$$

$$k = \frac{mg}{d}$$

$$k = \frac{mg}{d} = \frac{(0.55\ \text{kg})(9.80\ \text{m/s}^2)}{2.0 \times 10^{-2}\ \text{m}} = 2.7 \times 10^2\ \text{N/m}$$

7.4 KINETIC ENERGY AND THE WORK – KINETIC ENERGY THEOREM

- Kinetic Energy is the energy of a particle due to its motion
  - $K = ½ mv^2$
    - K is the kinetic energy
    - m is the mass of the particle
    - v is the speed of the particle



- The Work-Kinetic Energy Principle states
- $\Sigma W = \Delta K = K_f - K_i$

$$\sum W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

- ***Example 7.4 :***

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force

of 12 N. Find the speed of the block after it has moved 3.0 m.

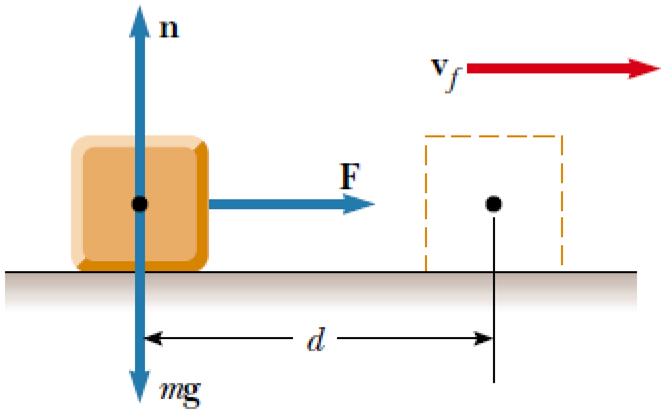
- ***Solution:***

$$W = Fd = (12\text{ N})(3.0\text{ m}) = 36\text{ N}\cdot\text{m} = 36\text{ J}$$

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - 0$$

$$v_f^2 = \frac{2W}{m} = \frac{2(36\text{ J})}{6.0\text{ kg}} = 12\text{ m}^2/\text{s}^2$$

$$v_f = 3.5\text{ m/s}$$



- ***Example 7.4 :***
- A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1.0 x 10^3 N/m. The spring is compressed 2.0 cm and is then released from rest. Calculate the speed of the block as it passes through the equilibrium position *x* = 0 if the surface is frictionless.
- ***Solution:***

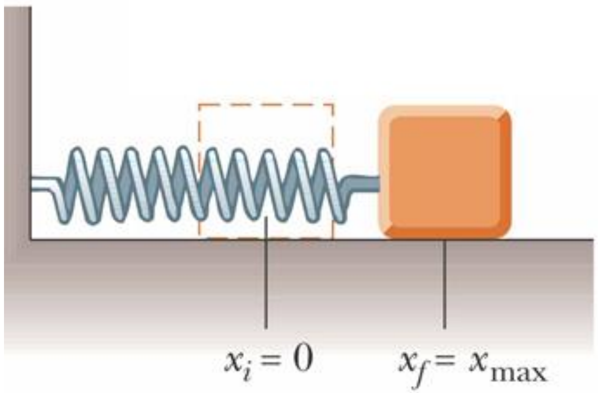
$$W_s = \frac{1}{2}kx_{\max}^2 = \frac{1}{2}(1.0 \times 10^3\ \mathrm{N/m})(-2.0 \times 10^{-2}\ \mathrm{m})^2 = 0.20\ \mathrm{J}$$

$$W_s = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2}$$

$$0.20\ \mathrm{J} = \frac{1}{2}(1.6\ \mathrm{kg})\,v_f^{\,2} - 0$$

$$v_f^{\,2} = \frac{0.40\ \mathrm{J}}{1.6\ \mathrm{kg}} = 0.25\ \mathrm{m^2/s^2}$$

$$v_f = \quad 0.50\ \mathrm{m/s}$$



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Problems

- Problem (1) :
- Solution: