The Laws of Motion

Chapter 5

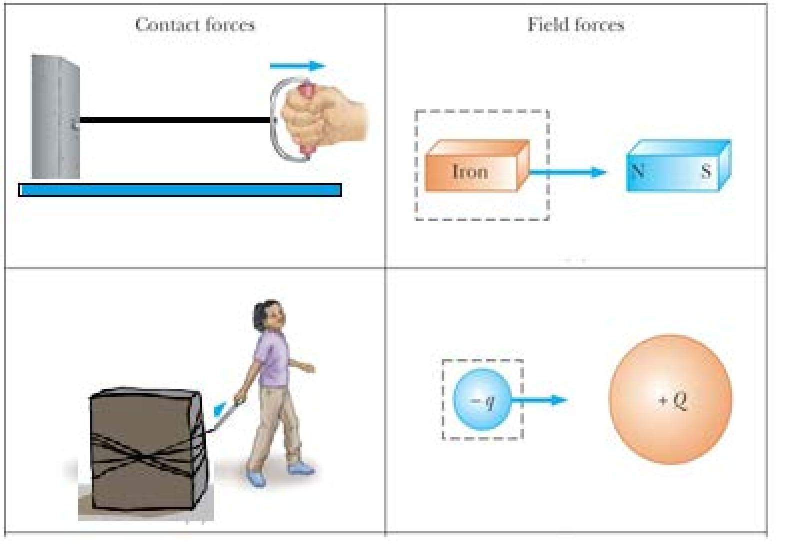
5.1 The Concept of Force

- **A force** is that which causes an acceleration
- **The *net force*** is the vector sum of all the forces acting on an object
- **When the net force is equal to zero**:
  - The acceleration is equal to zero
  - The velocity is constant
- ***Equilibrium*** occurs when the net force is zero
  - The object, if at rest, will remain at rest
  - If the object is moving, it will continue to move at a constant velocity

Classes of Forces

- **Contact forces** involve physical contact between two objects
- **Field forces** act through empty space
  - No physical contact is required
- **Forces are vectors**,

  So you must use the rules for vector addition to find the net force acting on an object



Fundamental Forces

- **Gravitational force**
  – Between two objects
- **Electromagnetic forces**
  – Between two charges
- **Nuclear force**
  – Between subatomic particles
- **Weak forces**
  – Arise in certain radioactive decay processes

5.2 Newton's First Law and Inertial Frames

- In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity
  - This is also called the *law of inertia*
  - Newton's First Law describes what happens in the absence of a force
  - Also tells us that when no force acts on an object, the acceleration of the object is zero

- Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame
- A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame

5.3 Mass

- *inertia* is The tendency of an object to resist any attempt to change its velocity
- *Mass* is that property of an object that specifies how much resistance an object exhibits to changes in its velocity
- **Mass and weight** are two different quantities
- Weight is equal to the magnitude of the gravitational force exerted on the object

- Mass is an inherent property of an object
- Mass is independent of the object’s surroundings
- Mass is independent of the method used to measure it
- Mass is a scalar quantity
- The SI unit of mass is kg

5.4 Newton's Second Law

- When viewed from an inertial frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass
  - Force is the cause of change in motion, as measured by the acceleration
- Algebraically,

$$\sum \mathbf{F} = m\,\mathbf{a}$$

- $\Sigma\mathbf{F}$ is the net force
  - This is the vector sum of all the forces acting on the object
- Newton's Second Law can be expressed in terms of components:
  - $\Sigma F_x = m\, a_x$
  - $\Sigma F_y = m\, a_y$
  - $\Sigma F_z = m\, a_z$

System of Units	Mass	Acceleration	Force
SI	kg	m/s^2	$N = kg \cdot m/s^2$

5.5 The Gravitational Force and Weight

- The gravitational force, $\mathbf{F}_g$, is the force that the earth exerts on an object
- This force is directed toward the center of the earth
- Its magnitude is called the weight of the object
- Weight = $|\mathbf{F}_g| = mg$

- Because it is dependent on g, the weight varies with location
  - g, and therefore the weight, is less at higher altitudes
- Weight is not an inherent property of the object

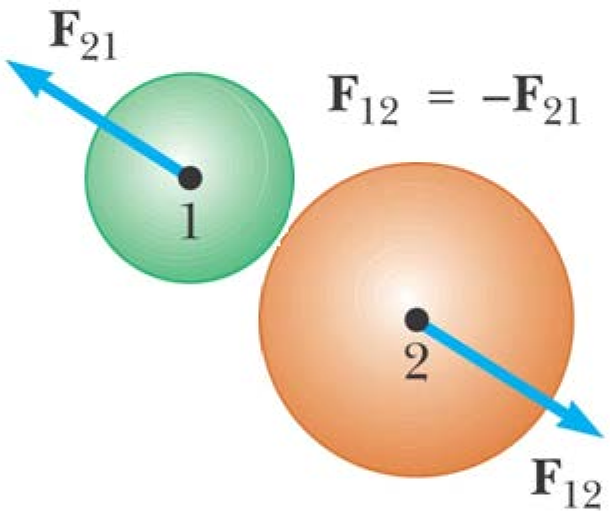
5.6 Newton's Third Law

- Forces always occur in pairs
- A single isolated force cannot exist
- The action force is equal in magnitude to the reaction force and opposite in direction
  - One of the forces is the action force, the other is the reaction force
  - It doesn't matter which is considered the action and which the reaction
  - The action and reaction forces must act on different objects and be of the same type

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

Action-Reaction Examples, 1

- The force $\mathbf{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to $\mathbf{F}_{21}$ exerted by object 2 on object 1
- $\mathbf{F}_{12} = - \mathbf{F}_{21}$



5.7 Some Applications of Newton's Laws

- Assumptions
  - Objects can be represented as particles
  - Masses of strings or ropes are negligible
  - Interested only in the external forces acting on the object
  - Initially dealing with frictionless surfaces

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Objects in Equilibrium

- If the acceleration of an object that can be modeled as a particle is zero, the object is said to be in **equilibrium**
- Mathematically, the net force acting on the object is zero

$$\sum F = 0$$

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

Objects in Equilibrium

- Exmple(1)

  - the force of gravity (**F**$_g$)
  - the tension in the chain (**T**)

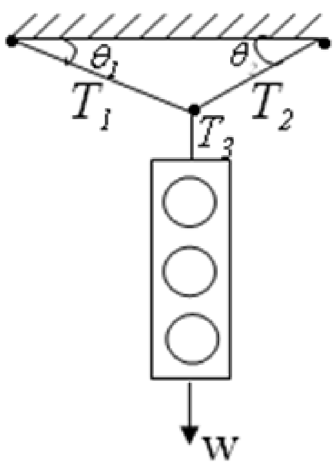
$$\sum F_y = 0 \rightarrow T - F_g = 0 \qquad T = F_g$$

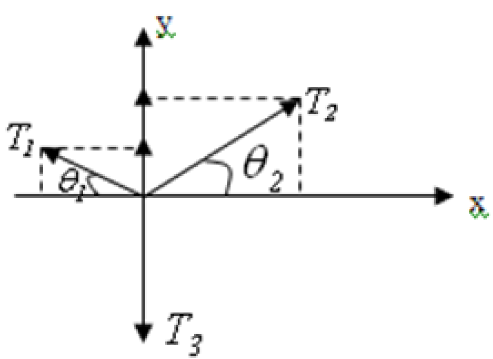
Objects in Equilibrium

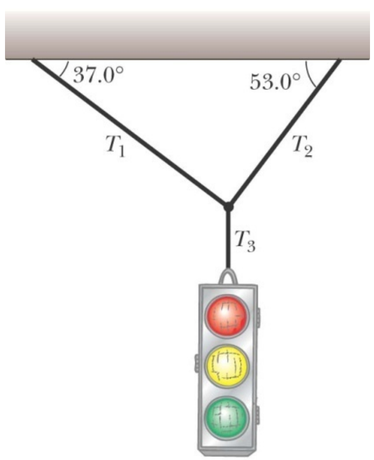
- Exmple(2)

$$\Sigma F_x = T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$$

$$\Sigma F_y = T_1 \sin\theta_1 + T_2 \sin\theta_2 - w = 0$$







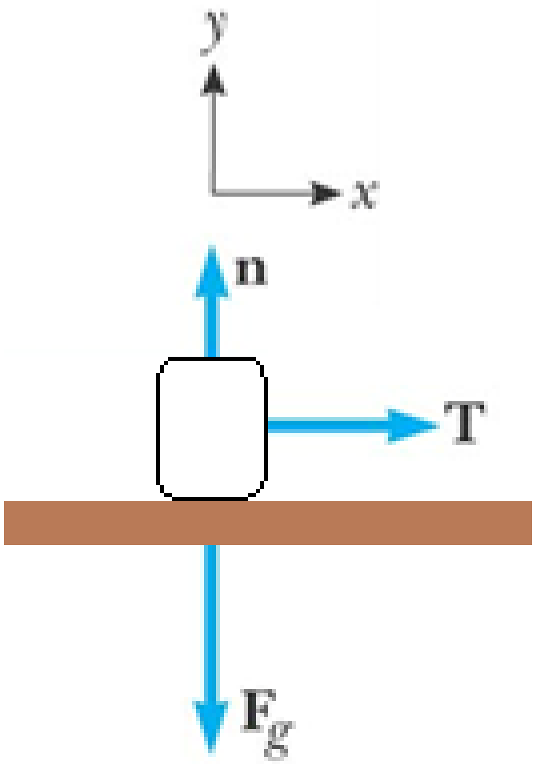
(a)

Objects Experiencing a Net Force

- Forces acting on the crate:
  - A tension, the magnitude of force **T**
  - The gravitational force, $\mathbf{F}_g$
  - The normal force, **n**, exerted by the floor

$$\sum F_x = T = ma_x$$

$$\sum F_y = n - F_g = 0 \rightarrow n = F_g$$



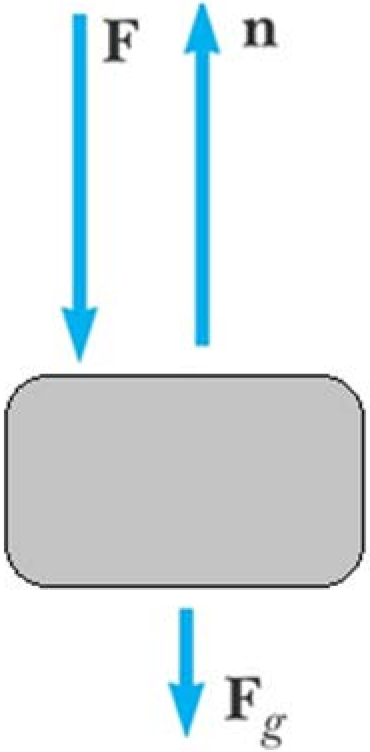
Note About the Normal Force

- The normal force is **not** always equal to the gravitational force of the object
- For example, in this case

$$\sum F_y = n - F_g - F = 0$$

and $n = F_g + F$

- **n** may also be less than $\mathbf{F}_g$



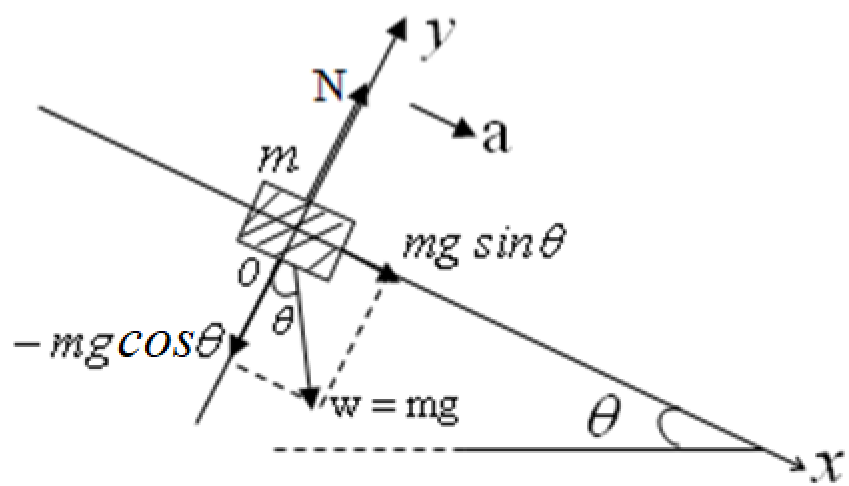
Inclined Planes

- The motion is on an inclined plane, and the "acceleration" - in this case - is $a_x = a$ and $a_y = 0$.

$$\Sigma \mathbf{F}_x = m\mathbf{g}\sin\theta = m\,\mathbf{a}_x$$

$$\Sigma \mathbf{F}_y = \mathbf{N} - m\mathbf{g}\cos\theta = 0$$

$$\mathbf{a}_x = \mathbf{g}\sin\theta \qquad \mathbf{N} = m\mathbf{g}\cos\theta$$

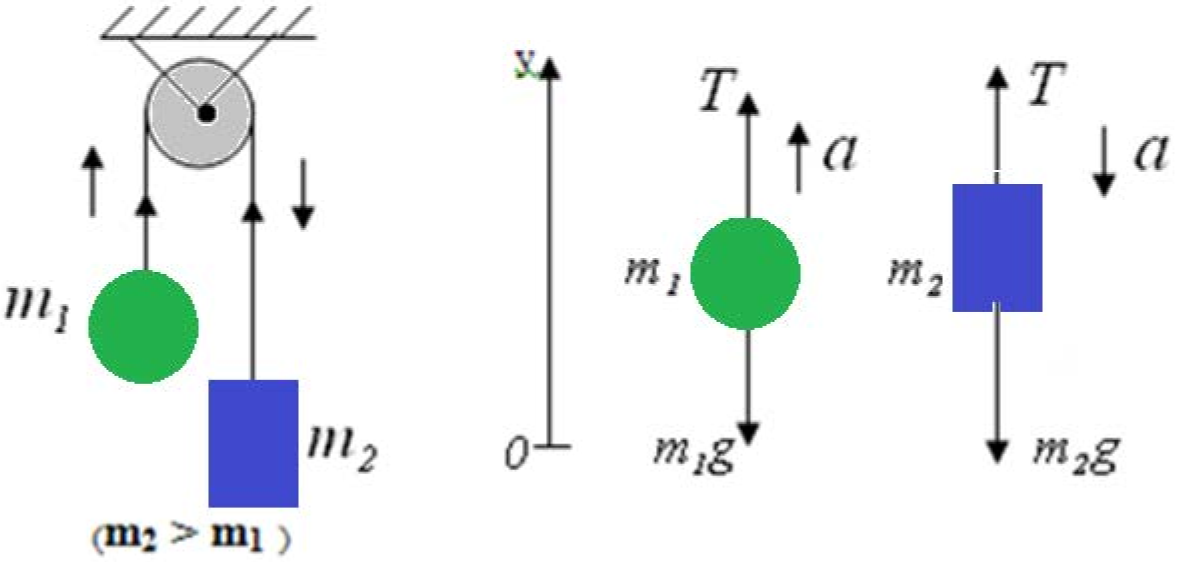


Multiple Objects

- It consists of two objects (m_1, m_2) connected by a rope passing on a smooth pulley

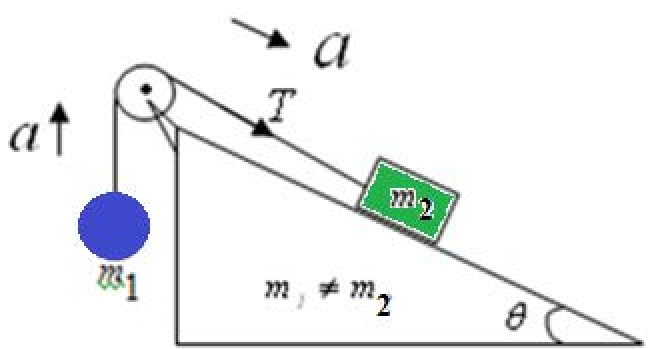
$$\Sigma \mathbf{F}_y = \mathbf{T} - m_1 \mathbf{g} = m_1 \mathbf{a}$$

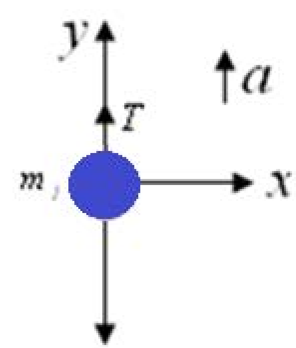
$$\Sigma \mathbf{F}_y = \mathbf{T} - m_2 \mathbf{g} = -m_2 \mathbf{a}$$

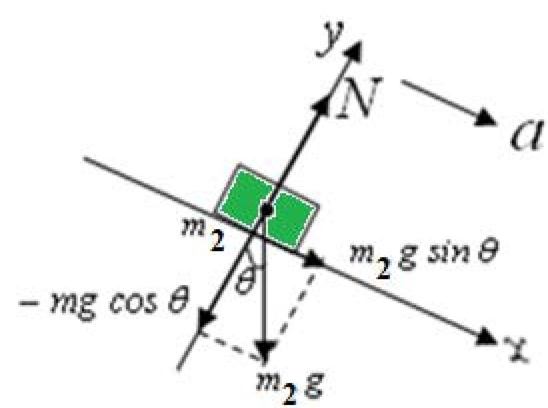


Inclined Planes and Multiple Objects

- In which the motion on an inclined frictionless surface is combined with vertical motion
- by applying Newton's second law to mass m1, we get
  - $\sum \mathbf{F}_x = 0$, $\sum \mathbf{F}_y = \mathbf{T} - m_1\mathbf{g} = m_1\mathbf{a}$, $\mathbf{T} > m_1\mathbf{g}$
- For mass m2, we get
  - $\sum \mathbf{F}_x = m_2\mathbf{g}\sin\theta - \mathbf{T} = m_2\mathbf{a}$
  - $\sum \mathbf{F}_y = \mathbf{N} - m_2\,\mathbf{g}\cos\theta = 0$







5.8 Forces of Friction

- When an object is in motion on a surface, there will be a resistance to the motion
- This resistance is called the *force of friction*
- Friction is proportional to the normal force

$$f_s \leq \mu_s n \quad \text{and} \quad f_k = \mu_k n$$

- The coefficient of friction (μ) depends on the surfaces in contact

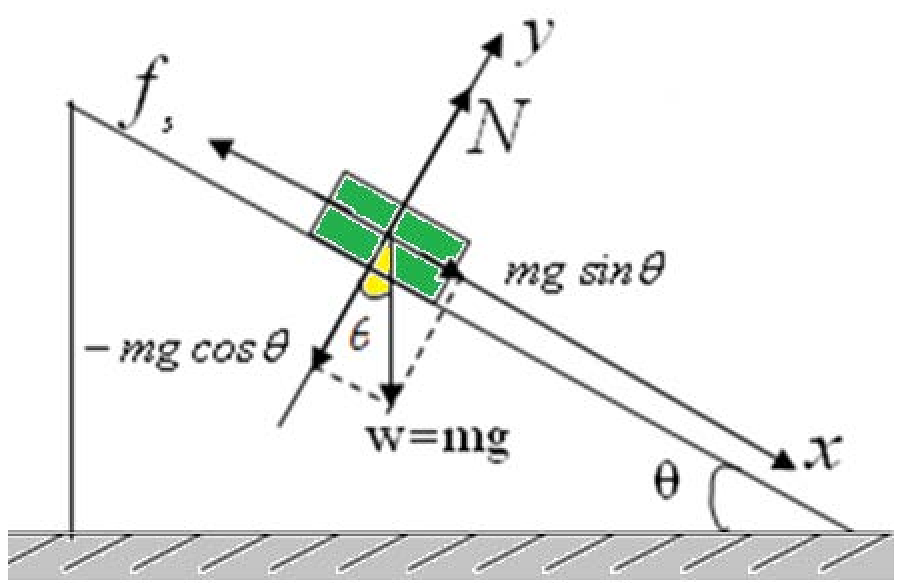
Friction in Newton's Laws Problems

- The block is sliding down the plane, so friction acts up the plane
  - For μ_s, use the angle where the block just slips
  - For μ_k, use the angle where the block slides down at a constant speed

$$\Sigma \mathbf{F}x = m\mathbf{g} \sin\theta - \mathbf{f}_s = 0$$

$$\Sigma \mathbf{F}y = \mathbf{N} - m\mathbf{g} \cos\theta = 0$$

$$\mu = \tan\theta$$



The strategy of applying Newton's laws

To use Newton's laws in most applications, the student must follow the following steps:

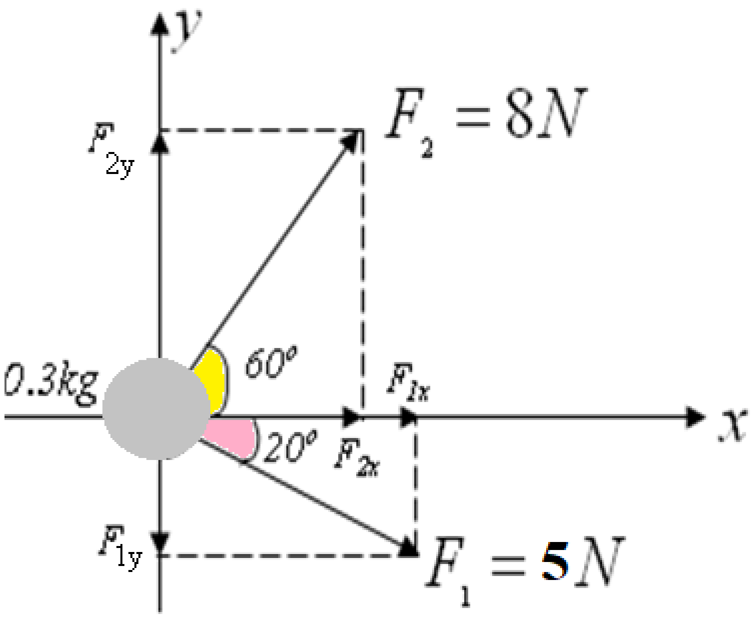
1- Determining the (x, y) **axes** to describe the movement, provided that the (x) axis is in the direction of movement
2- Draw a **diagram** of the distribution of forces acting on the body
3 - Decomposition of all forces into **components** in the direction of the axes (x, y)
4 - Applying **Newton's second law** to the x-axis and y-axis

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y$$

5 - You get a set of equations. By **solving** these equations simultaneously, you get what you want.

Problems

- Problem (1) :
- Find the magnitude and direction of the final acceleration of the body shown in the following figure.
- Solution:



$\Sigma F_x = F_{1x} + F_{2x}$

$= F_1 \cos 20^o + F_2 \cos 60^o$

$= (5 \times 0.94) + (8 \times 0.5) = 8.$

$\Sigma F_y = F_{1y} + F_{2y} = - F_1 \sin 20^o + F_2 \sin 60^o$

$= (- 5 \times 0.342) + (8 \times 0.866) = 5.2N$

$$\mathbf{a}_x = \frac{\sum \mathbf{F}_x}{m} = \frac{8.7}{0.3} = 29 \ m/s^2 \qquad \mathbf{a}_y = \frac{\sum \mathbf{F}_y}{m} = \frac{5.2}{0.3} = 17 \ m/s^2$$

$$\mathbf{a} = \sqrt{\mathbf{a}_x^2 + \mathbf{a}_y^2} = \sqrt{(29)^2 + (17)^2} = 34 \ m/s^2 \qquad \theta = \tan^{-1}\left(\frac{\mathbf{a}_y}{\mathbf{a}_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 31^o$$

- Problem (2) :
- A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as in Figure 5.10a. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?
- Solution:

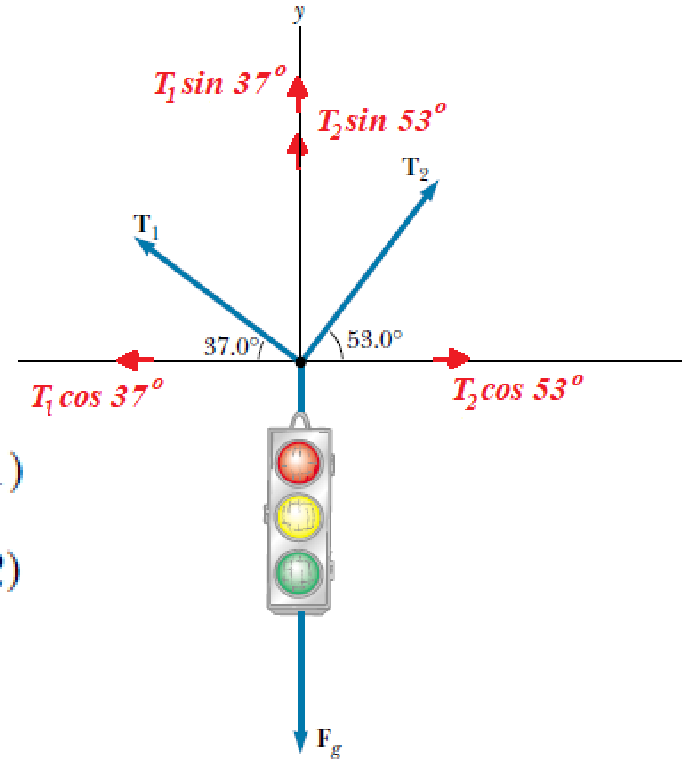
1- Choose the coordinate axes shown in Figure

2- Resolve the forces acting on the knot into their components:

3- Knowing that the knot is in equilibrium ($a = 0$)

4- Apply Newton's law in x- and y- coordinates

Force	x Component	y Component
$\mathbf{T}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\mathbf{T}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\mathbf{F}_g$	0	-125 N



$$\sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0 \quad (1)$$

$$\sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-125\ N) = 0 \quad (2)$$

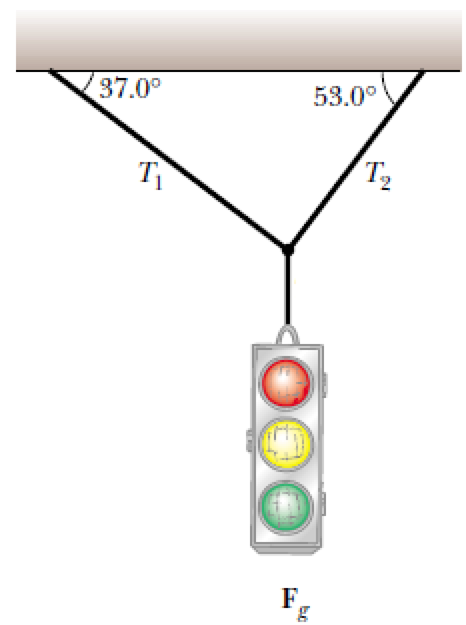
We solve eq. (1) for *T_2 in terms of T_1 to* obtain

$$T_2 = T_1\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 1.33\,T_1$$

Substituyte for *T_2 in eq. (2) , we obtain*

$$T_1 \sin 37.0° + (1.33\,T_1)(\sin 53.0°) - 125\ \text{N} = 0$$

$$T_1 = 75.1\ \text{N} \quad \textit{and} \quad T_2 = 1.33\,T_1 = 99.9\ \text{N}$$



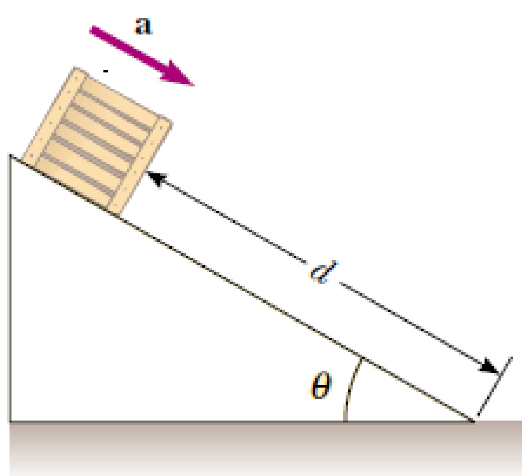
- Problem (3) :

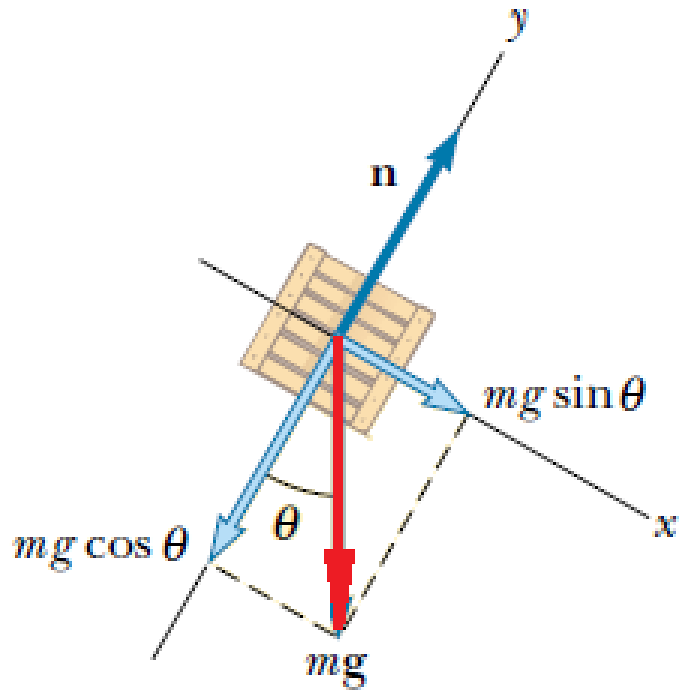
A crate of mass ***m*** is placed on a frictionless inclined plane of angle ***θ***

(a) Determine the acceleration of the crate after it is released.

(b) Suppose the crate is released from rest at the top of the incline, and the distance from the front edge of the crate to the bottom is ***d***. How long does it take the front edge to reach the bottom, and what is its speed just as it gets there?

- Solution:





After choosing the coordinates , resolving the force of gravity and applying Newton's law, we get

$$\sum F_x = mg\sin\theta = ma_x \qquad (1)$$

$$\sum F_y = n - mg\cos\theta = 0 \qquad (2)$$

Note that *a_y=0 , from eq. (1) we obtain :*

$$a_x = g\sin\theta$$

Because a_x *constant, by applying equations of motion:*

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

With the displacement (*x_f - x_i =d) and v_{xi} =0, we obtain*

$$d = \frac{1}{2}a_x t^2 \quad \longrightarrow \quad t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g\sin\theta}}$$

To calculate the final velocity , we use the following eq.

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

With *$v_{xi} = 0$, we find that :*

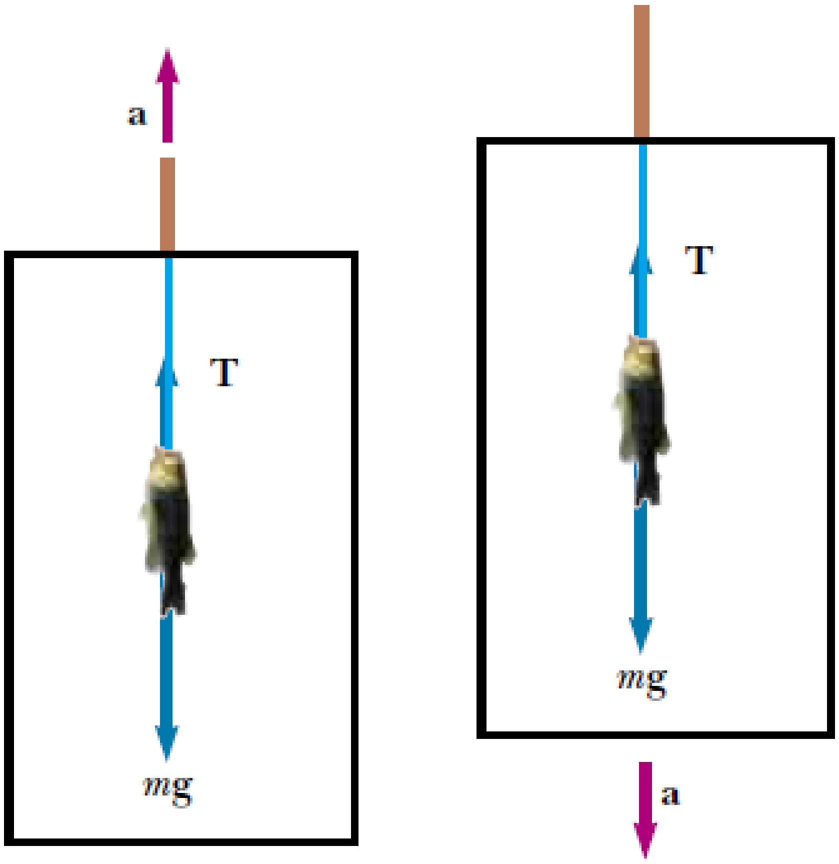
$$v_{xf}^2 = 2a_x d \quad \Rightarrow \quad v_{xf} = \sqrt{2a_x d} = \sqrt{2gd\sin\theta}$$

- Problem (4) :
- A person weighs a fish of mass *m on a spring scale attached to*
- the ceiling of an elevator, as illustrated in Figure. Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.
- Solution:

Newton's second law applied to the fish gives the net force on the fish:

$$\sum F_y = T - mg = ma_y$$

So that *T* in equation is the apparent weight of the fish



For example, if the weight of the fish is 40.0 N and a is upward, so that a = 2 m/s2 ,

$$T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right)$$

$$= (40.0\ \mathrm{N})\left(\frac{2.00\ \mathrm{m/s^2}}{9.80\ \mathrm{m/s^2}} + 1\right)$$

$$= \quad 48.2\ \mathrm{N}$$

If the weight of the fish is 40.0 N and ***a*** is downward,

so that a = -2 m/s^2 ,

$$T = mg\left(\frac{a_y}{g} + 1\right)$$

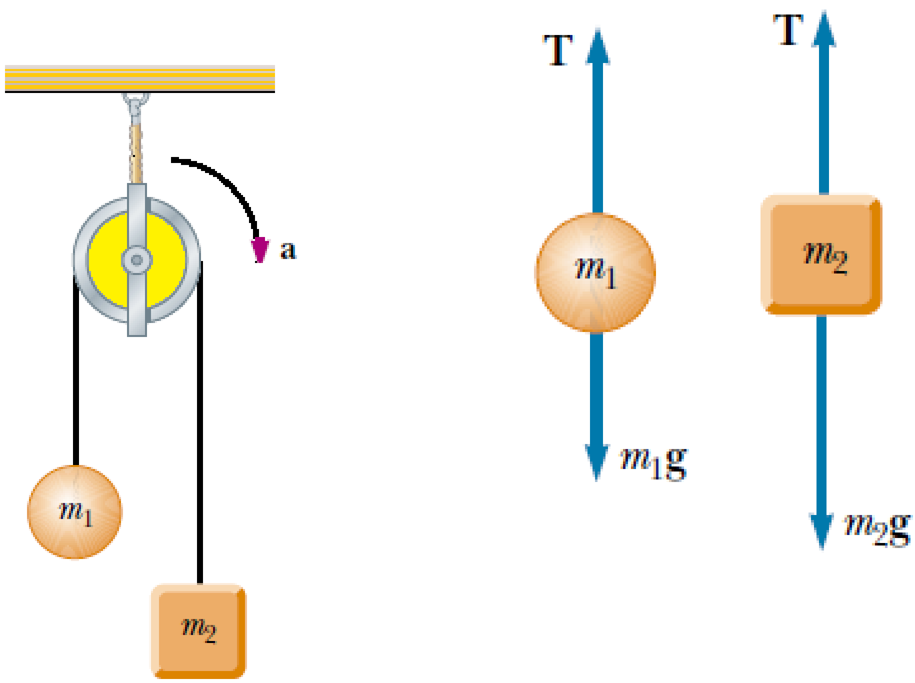
$$= (40.0\ \mathrm{N})\left(\frac{-2.00\ \mathrm{m/s^2}}{9.80\ \mathrm{m/s^2}} + 1\right)$$

$$= \quad 31.8\ \mathrm{N}$$

- Problem (5) :

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, as shown in Figure. Determine the magnitude of the acceleration of the two objects and the tension in the cord.

- Solution:



When Newton's second law is applied to object 1, we Obtain :

$$\sum F_y = T - m_1 g = m_1 a_y \qquad (1)$$

for object 2 we find

$$\sum F_y = m_2 g - T = m_2 a_y \qquad (2)$$

When (2) is added to (1),

$$-m_1 g + m_2 g = m_1 a_y + m_2 a_y$$

When (3) is substituted into (1), we obtain

$$a_y = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g \qquad (3)$$

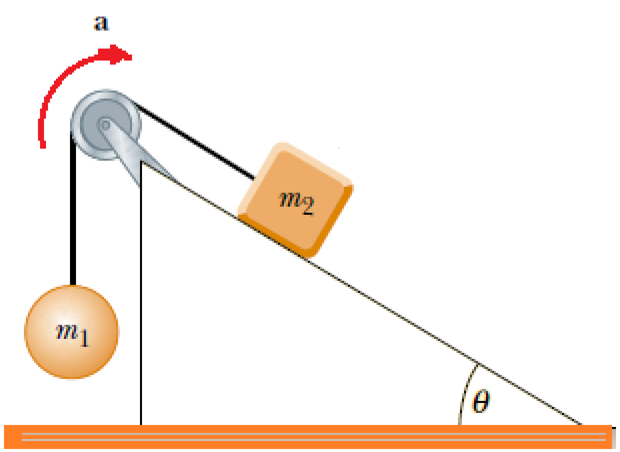
$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2} \right) g \qquad (4)$$

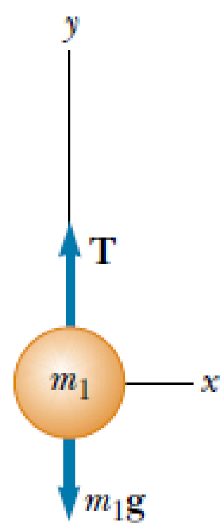
- for $m_1 = 2.00\ kg$ and $m_2 = 4.00\ kg.$
- a_y $3.27\ m/s^2$, T $26.1\ N.$

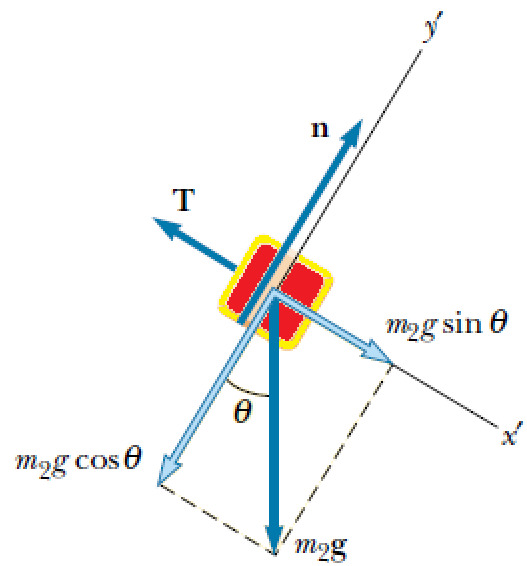
- Problem (6) :

A ball of mass m_1 *and a block of mass* m_2 *are attached by a* cord that passes over a frictionless pulley . The block lies on a frictionless incline of angle $\boldsymbol{\theta}$. Find the magnitude of the acceleration of the two objects and the tension in the cord.

- Solution:







with the choice of the upward direction as positive for the ball :

$$\sum F_x = 0 \qquad (1)$$

$$\sum F_y = T - m_1 g = m_1 a_y = m_1 a \qquad (2)$$

For the block it is convenient to choose the positive x-axis along the incline

$$\sum F_{x'} = m_2 g \sin\theta - T = m_2 a_{x'} = m_2 a \qquad (3)$$

$$\sum F_{y'} = n - m_2 g \cos\theta = 0 \qquad (4)$$

if we solve (2) for *T and then* substitute this value for *T into (3) and solve for **a**, we obtain*

$$a = \frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2} \qquad (5)$$

$$T = \frac{m_1 m_2 g(\sin\theta + 1)}{m_1 + m_2} \qquad (6)$$

If $m_1 = 10$ *kg*, $m_2 = 5$ *kg, and* $45.0°$*,* the acceleration $a = 4.22\ m/s^2$*,* *where the negative sign indicates* that the block accelerates up the incline and the ball accelerates downward.

- Problem (7) :

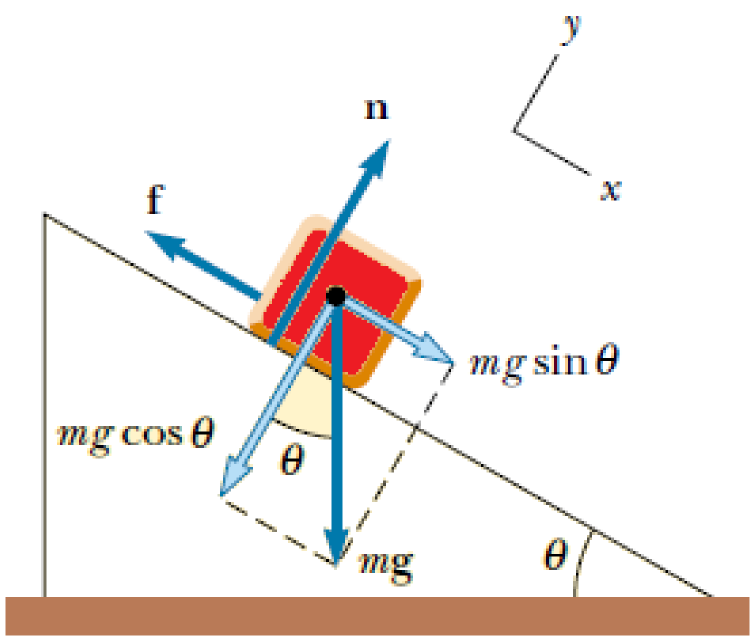
The following is a simple method of measuring coefficients of friction: Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in Figure. The incline angle is increased until the block starts to move. Let us show that by measuring the critical angle $\boldsymbol{\theta_c}$ *at which this* slipping just occurs, we can obtain $\boldsymbol{\mu_s}$ *.*

Solution:

Newton's second law applied to the block for this balanced situation gives

$$\sum F_x = mg\sin\theta - f_s = ma_x = 0 \quad (1)$$

$$\sum F_y = n - mg\cos\theta = ma_y = 0 \quad (2)$$



We can eliminate *mg by substituting* $mg = n/\cos\theta$ *from* (2) into (1) to get

$$f_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = n\tan\theta$$

When the incline is at the critical angle $\boldsymbol{\theta_c}$, *we know that* $\boldsymbol{f_s = f_{s,max} = \mu_s n}$, *and so at this angle, (3) becomes*

$$\mu_s n = n\tan\theta_c$$

$$\mu_s = \tan\theta_c$$

For example, if the block just slips at $\boldsymbol{\theta_c} = 20°$, *then we find* that $\boldsymbol{\mu_s} = \tan 20° = 0.364.$

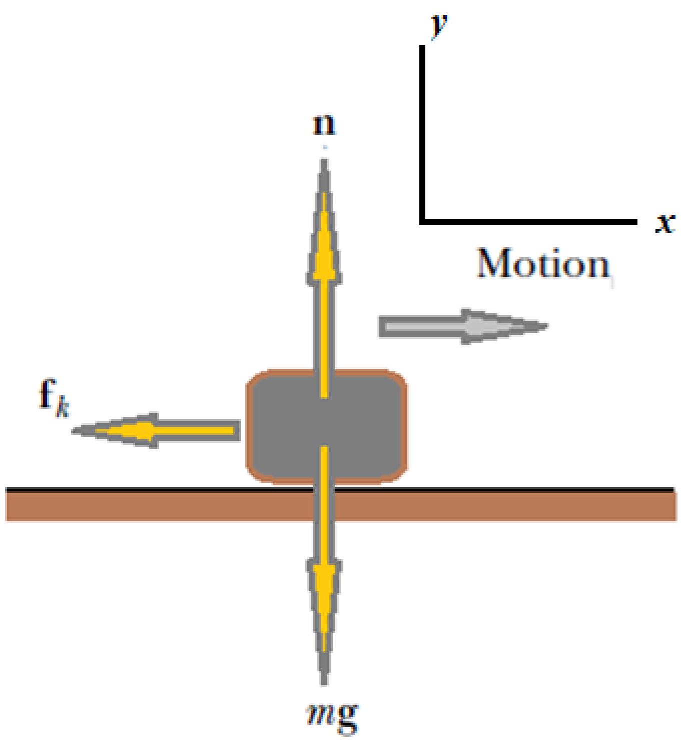
Once the block starts to move at $\theta \geq \theta_c$, *it accelerates* down the incline and the force of friction is $f_k = \mu_k n$.
it may be possible to find an angle θ'_c such that the block moves down the incline with constant speed ($a_x = 0$). *In this case,*

$$\mu_k = \tan \theta'_c$$

- Problem (8) :

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

Solution:



we apply Newton's second law in both directions x and y , we get

$$\sum F_x = -f_k = ma_x \qquad (1)$$

$$\sum F_y = n - mg = 0 \qquad (a_y = 0) \qquad (2)$$

But $\boldsymbol{f_k = \mu_k n}$*, and from (2) we see that* $\boldsymbol{n = mg}$*. Therefore,* (1) becomes

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

The negative sign means the acceleration is to the left; this means that the puck is slowing down.

Because the acceleration is constant, we can use the following Equation

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i),$$

with $x_i = 0$ and $v_{xf} = 0$ and $a_x = -\mu_k g$

$$v_{xi}^2 + 2ax_f = 0$$

$$v_{xi}^2 - 2\mu_k g x_f = 0 \quad \Rightarrow \quad \mu_k = \frac{v_{xi}^2}{2gx_f}$$

$$\mu_k = \frac{(20\ \mathrm{m/s})^2}{2(9.80\ \mathrm{m/s^2})(115\ \mathrm{m})} = 0.177$$