MATH 151

Mathematical Induction

Lecture 3

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Exercise 1: Use mathematical induction to show that the number $5n^2 - 3n$ is even for all integers $n \geq 0$.

Exercise 2: Use induction to show that $n^2 - 3n + 5$ is an odd integer for all $n \geq 2$

Exercise 3: Use mathematical induction to show that

$$1\,.\,2+2\,.\,3+3\,.\,4+...+n.(n+1)=\frac{n(n+1)(n+2)}{3} \text{ for } n\geq 1 \;(\,n \text{ is integer})$$

Exercise 4: Prove that $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)...\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$ for all integers $n\geq 2$.

Exercise 5: Prove that $3^{n-1} \geq 2^n + 1$ for all integers $n \geq 3$.

Exercise 6: Use mathematical induction to show that $n! > 2^{n+1}$ for all integers $n \geq 5$.

Exercise 7: Prove that $3/\left(4^n+2\right)$ for all integers $n \geq 0$

Exercise 8: Use mathematical induction to show that $2^n \geq n+12$ for all integers $n \geq 4$.

Exercise 9: Use mathematical induction to show that

$1.2^1 + 2.2^2 + 3.2^3 + ... + n.2^n = 2 + (n-1)2^{n+1}$ for all integers $n \geq 1$.

Exercise 10: Prove that

$$1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2(n+1)^2}{4} \text{ for all integers } n \geq 1$$

Exercise 11: Use mathematical induction to show that $n^2 - 6n + 8 \geq 0$ for all integers $n \geq 4$.

Exercise 12: Use mathematical induction to show that

$$1 + 4 + 7 + ... + (3n - 2) = \frac{n(3n-1)}{2} \text{ for all integers } n \geq 1.$$

Exercise 13: Prove that

$$1+2+2^2+2^3+...+2^n=2^{n+1}-1 \text{ for all integers } n\geq 0$$

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Exercise 14: Let $\{u_n\}$ be a sequence defined by the equations $u_1=0, u_2=1$ and $u_{n+1}=3u_n-2u_{n-1}-1$ for $n=2,3,4,...$ show that $u_n=n-1$ for all $n\geq 1$.

Exercise 15: Let $\{u_n\}$ be a sequence defined by the equations $u_0 = 12, u_1 = 21$ and $u_{n+1} = \frac{(u_n)^2 u_{n-1}}{9}$ for $n = 1,2,3,...$ show that u_n is an integer divisible by 3 for all $n \geq 0$.

Exercise 16: Let $\{u_n\}$ be a sequence defined by the equations $u_1 = 2, u_2 = 5$ and $u_{n+1} = 2u_n - u_{n-1} + 2$ for $n = 2,3,4,...$ show that $u_n = n^2 + 1$ for all $n \geq 1$.

Exercise 17: Let $\{a_n\}$ be a sequence defined as $\begin{cases} a_0 = 2 \quad , \quad a_1 = 4 \\ a_n = 4a_{n-1} - 3a_{n-2} \ , \ \forall n \geq 2 \end{cases}$

show that $a_n = 1 + 3^n$ for all integers $n \geq 0$.

Exercise 18: Let $\{a_n\}$ be a sequence defined as $\begin{cases} a_0 = 1 \, , a_1 = 2 \, , \, a_2 = 3 \\ a_n = a_{n-1} + a_{n-2} + 2a_{n-3} \ , \ \forall n \geq 3 \end{cases}$

show that $a_n \leq 3^n$ for all integers $n \geq 0$.

Exercise 19: Let $\{u_n\}$ be a sequence defined by the equations $u_1 = 1, u_2 = 2, u_3 = 3$ and $u_n = \dfrac{u_{n-1} + u_{n-2} + u_{n-3}}{3}$ for all $n \geq 4$ show that $1 \leq u_n \leq 3$ for all $n \geq 1$.

Exercise 20: Let $\{u_n\}$ be a sequence defined by the equations $u_1 = 2, u_2 = 4$ and $u_n = \dfrac{2u_{n-1} + u_{n-2} + 8}{3}$ for all $n \geq 3$ show that $u_n = 2n$ for all $n \geq 1$.

Exercise 21: Let $\{a_n\}$ be a sequence defined as $\begin{cases} a_0 = 2 \quad , \quad a_1 = 5 \\ a_{n+1} = 5a_n - 4a_{n-1} \; , \; \forall n \geq 1 \end{cases}$

show that $a_n = 1 + 4^n$ for all integers $n \geq 0$.

Exercise 22: Let $\{a_n\}$ be a sequence defined as $\begin{cases} a_0 = 2 \quad , \quad a_1 = 5 \\ a_{n+1} = 5a_n - 6a_{n-1} \; , \; \forall n \geq 1 \end{cases}$

show that $a_n = 2^n + 3^n$ for all integers $n \geq 0$.

Exercise 23: Let $\{u_n\}$ be a sequence defined by the equations

$u_1 = 2, u_2 = 3, u_3 = 4$ and $u_{n+1} = \frac{1}{3}\left(u_n + u_{n-1} + u_{n-2}\right)$ for all $n \geq 3$ show that

$2 \leq u_n \leq 4$ for all $n \geq 0$.