Motion in One Dimension

Chapter 2

Kinematics

- Describes motion while ignoring the agents that caused the motion
- For now, will consider motion in one dimension
  - Along a straight line
- Will use the particle model
  - A particle is a point-like object, has mass but infinitesimal size

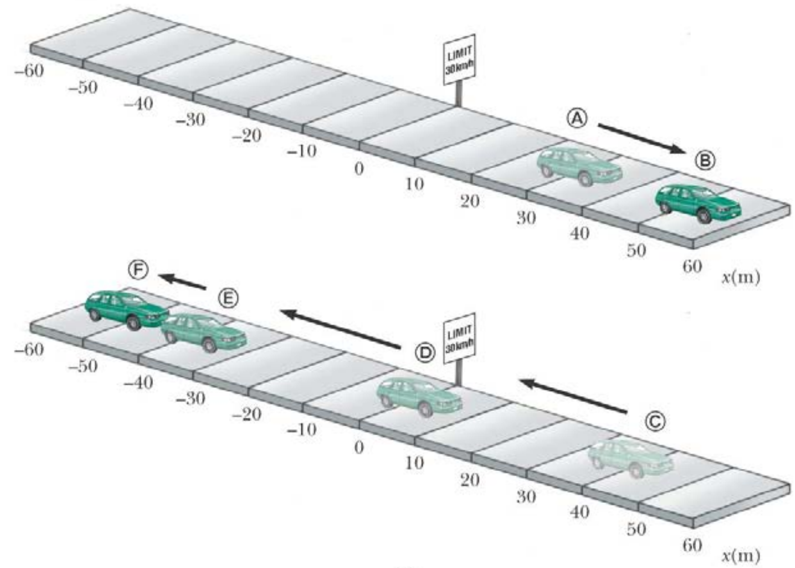
2.1 Position, Velocity, and Speed

Position

- Defined in terms of a **frame of reference**
  - One dimensional, so generally the x- or y-axis
- The object's position is its location with respect to the frame of reference

Position of the Car at Various Times

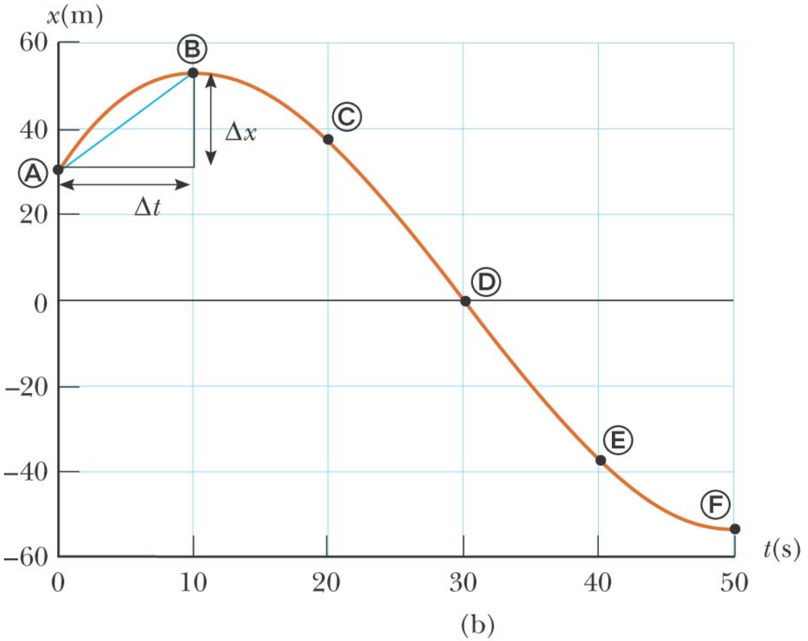
Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



(a)

Position-Time Graph

- The position-time graph shows the motion of the particle (car)
- The smooth curve is a guess as to what happened between the data points



Displacement

- Defined as the change in position during some time interval
  - Represented as Δx

$$\Delta x = x_f - x_i$$

  - SI units are meters (m) Δx can be positive or negative
- It is very important to recognize the difference between **displacement** and distance traveled. **Distance** is the length of a path followed by a particle.

Vectors and Scalars

- Vector quantities need both magnitude (size or numerical value) and direction to completely describe them
  - Will use + and – signs to indicate vector directions
- Scalar quantities are completely described by magnitude only
- Displacement is an example of a vector quantity

Average Velocity

- The **average velocity** is rate at which the displacement occurs

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

- The dimensions are length / time [L/T]
- The SI units are m/s
- Is also the slope of the line in the position – time graph

(if $x_f > x_i$), Δx is positive and $v_{x,\,avg} = \Delta x/\Delta t$ is positive.

(if $x_f < x_i$), Δx is negative and hence $v_{x,\,avg}$ is negative.

Average Speed

- Speed is a scalar quantity
  - same units as velocity
  - total distance / total time

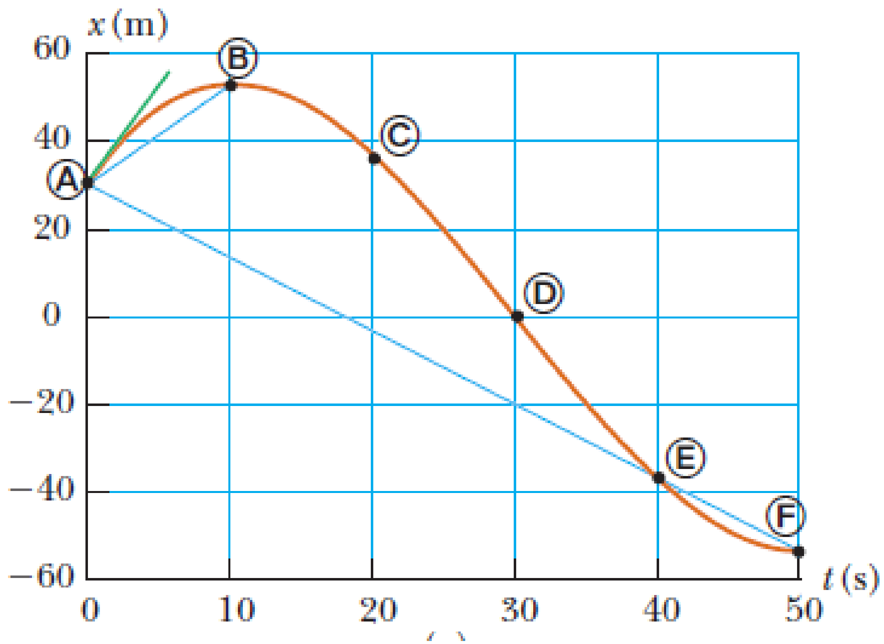
$$v_{avg} \equiv \frac{d}{\Delta t}$$

2.2 Instantaneous Velocity and Speed

- The general equation for instantaneous velocity is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The instantaneous velocity can be positive, negative, or zero



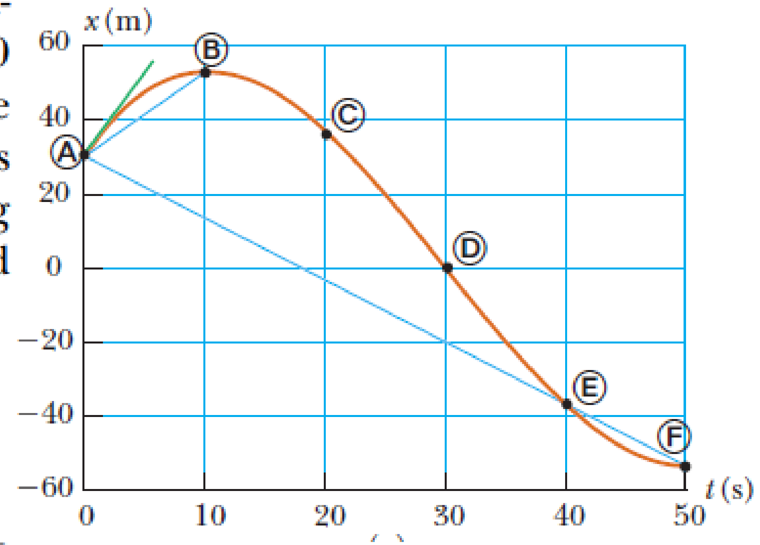
Instantaneous Speed

- The instantaneous speed is the magnitude of the instantaneous velocity
- Remember that the average speed is <u>not</u> the magnitude of the average velocity

EXAMPLE 2.1 Calculating the Variables of Motion

Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions Ⓐ and Ⓕ.

Solution The units of displacement must be meters, and the numerical result should be of the same order of magnitude as the given position data (which means probably not 10 or 100 times bigger or smaller). From the position–time graph given in Figure 2.1b, note that $x_A = 30$ m at $t_A = 0$ s and that $x_F = -53$ m at $t_F = 50$ s. Using these values along with the definition of displacement, Equation 2.1, we find that



$$\Delta x = x_F - x_A = -53 \text{ m} - 30 \text{ m} = -83 \text{ m}$$

This result means that the car ends up 83 m in the negative direction (to the left, in this case) from where it started. This number has the correct units and is of the same order of magnitude as the supplied data. A quick look at Figure 2.1a indicates that this is the correct answer.

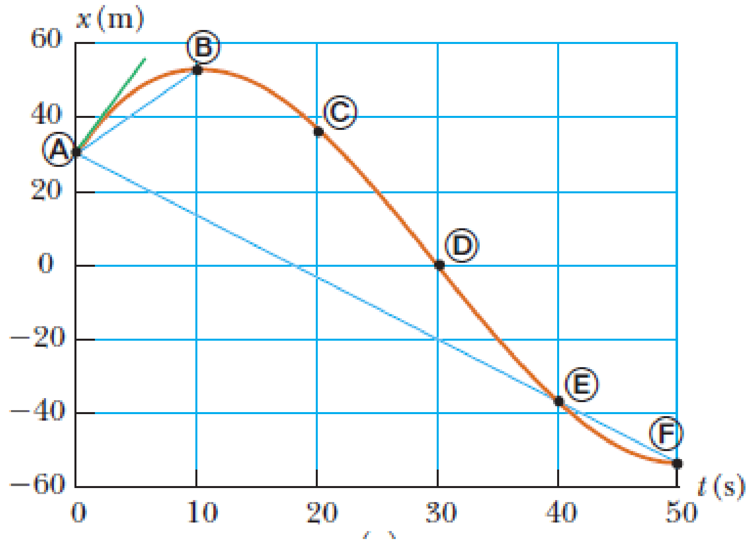
It is difficult to estimate the average velocity without completing the calculation, but we expect the units to be meters per second. Because the car ends up to the left of where we started taking data, we know the average velocity must be negative. From Equation 2.2,

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_{\text{F}} - x_{\text{A}}}{t_{\text{F}} - t_{\text{A}}}$$

$$= \frac{-53 \text{ m} - 30 \text{ m}}{50 \text{ s} - 0 \text{ s}} = \frac{-83 \text{ m}}{50 \text{ s}} = \boxed{-1.7 \text{ m/s}}$$

We find the car's average speed for this trip by adding the distances traveled and dividing by the total time:

$$\text{Average speed} = \frac{22 \text{ m} + 52 \text{ m} + 53 \text{ m}}{50 \text{ s}} = \boxed{2.5 \text{ m/s}}$$



2.3 Analysis Models: The Particle Under Constant Velocity

- If the velocity of a particle is constant, its instantaneous velocity at any instant during a time interval is the same as the average velocity.

$$v_x = v_{x,\,\text{avg}} \quad \text{and} \quad v_x = \frac{\Delta x}{\Delta t}$$

- The position of the particle is given by

$$x_f = x_i + v_x t \quad (\text{for constant } v_x)$$

Constant Speed

- the **particle under constant speed** is given by

$$v = \frac{d}{\Delta t}$$

- As an example, imagine a particle moving at a constant speed in a circular path. If the speed is 5.00 m/s and the radius of the path is 10.0 m, we can calculate the time interval required to complete one trip around the circle:

$$v = \frac{d}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{d}{v} = \frac{2\pi r}{v} = \frac{2\pi\ (10.0\ \text{m})}{5.00\ \text{m/s}} = 12.6\ \text{s}$$

2.4 Acceleration

- When the velocity of a particle changes with time, the particle is said to be *accelerating.*

- For example, the magnitude of the velocity of a car increases when you step on the gas and decreases when you apply the brakes. Let us see how to quantify acceleration.

Average Acceleration

- Acceleration is the rate of change of the velocity

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{\Delta t}$$

- Dimensions are L/T^2
- SI units are m/s^2

Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$$

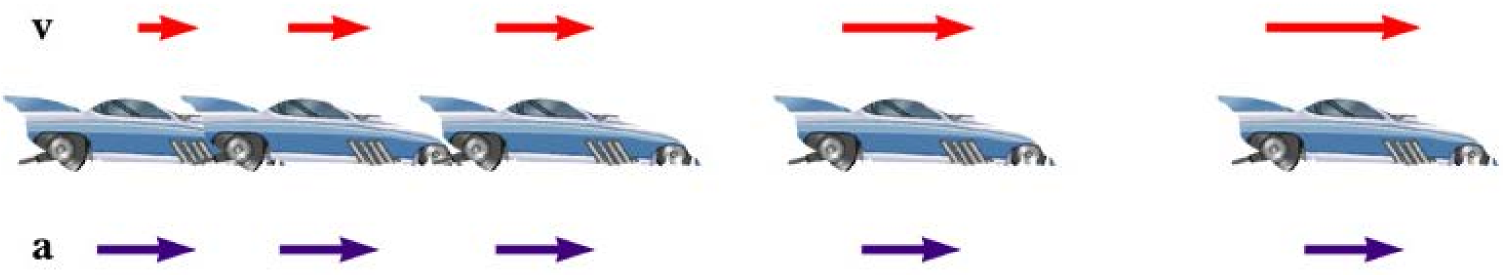
2.5 Motion Diagrams

- When an object's velocity and acceleration are in the same direction, the object is speeding up
- When an object's velocity and acceleration are in the opposite direction, the object is slowing down

Acceleration and Velocity, 1

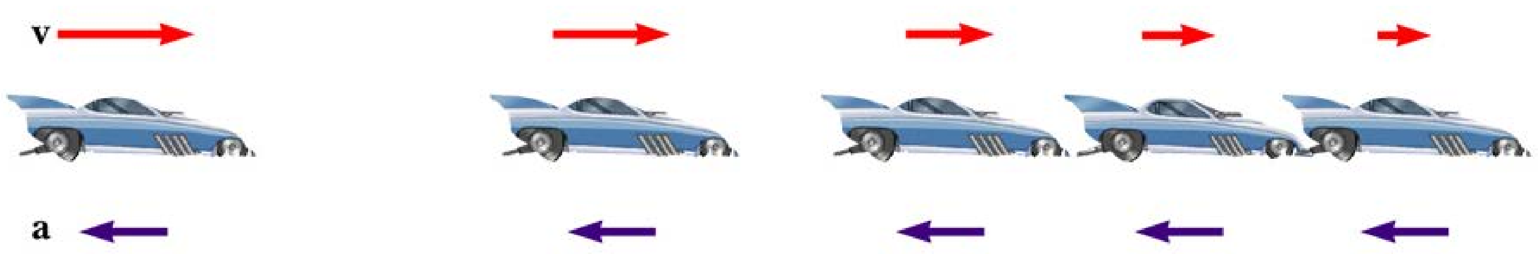
- The car is moving with constant positive velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

Acceleration and Velocity, 2



- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)
- This shows positive acceleration and positive velocity

Acceleration and Velocity, 3



- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)
- Positive velocity and negative acceleration

2.6 The Particle Under Constant Acceleration

$$v_{xf} = v_{xi} + a_x t$$

$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x (x_f - x_i)$$

Kinematic Equations

- The kinematic equations may be used to solve any problem involving one-dimensional motion with a constant acceleration
- You may need to use two of the equations to solve one problem
- Many times there is more than one way to solve a problem

Kinematic Equations, specific

- For constant *a*, $$v_{xf} = v_{xi} + a_x t$$
- Can determine an object's velocity at any time *t* when we know its initial velocity and its acceleration
- Does not give any information about displacement

Kinematic Equations, specific

- For constant acceleration,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

- Gives final position in terms of velocity and acceleration
- Doesn't tell you about final velocity

Kinematic Equations, specific

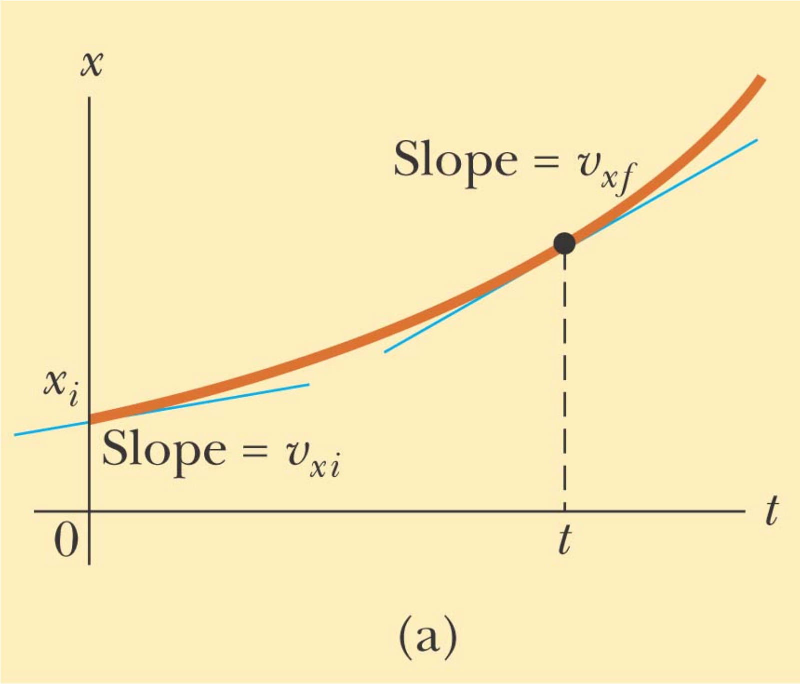
- For constant *a*,

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

- Gives final velocity in terms of acceleration and displacement
- Does not give any information about the time

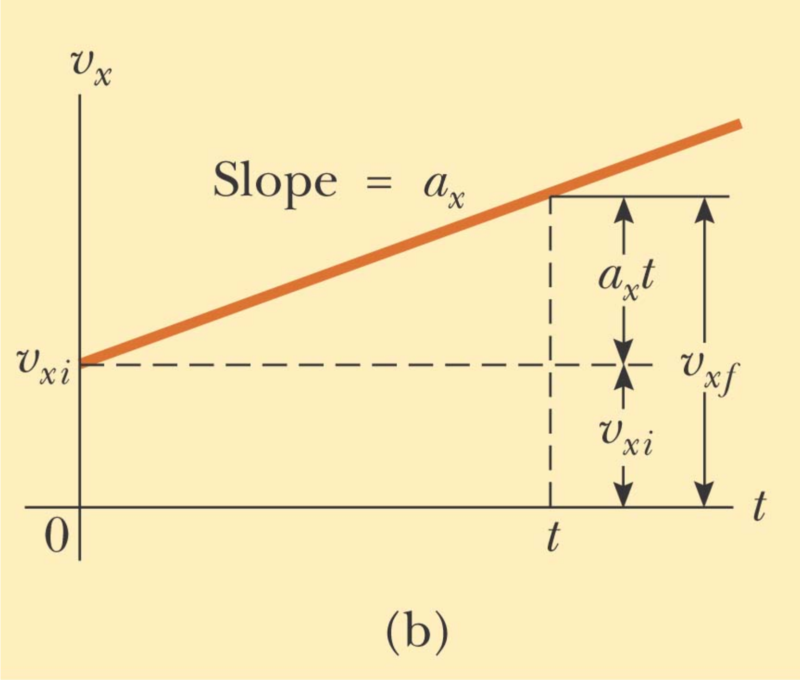
Graphical Look at Motion – displacement – time curve

- The slope of the curve is the velocity
- The curved line indicates the velocity is changing
  - Therefore, there is an acceleration



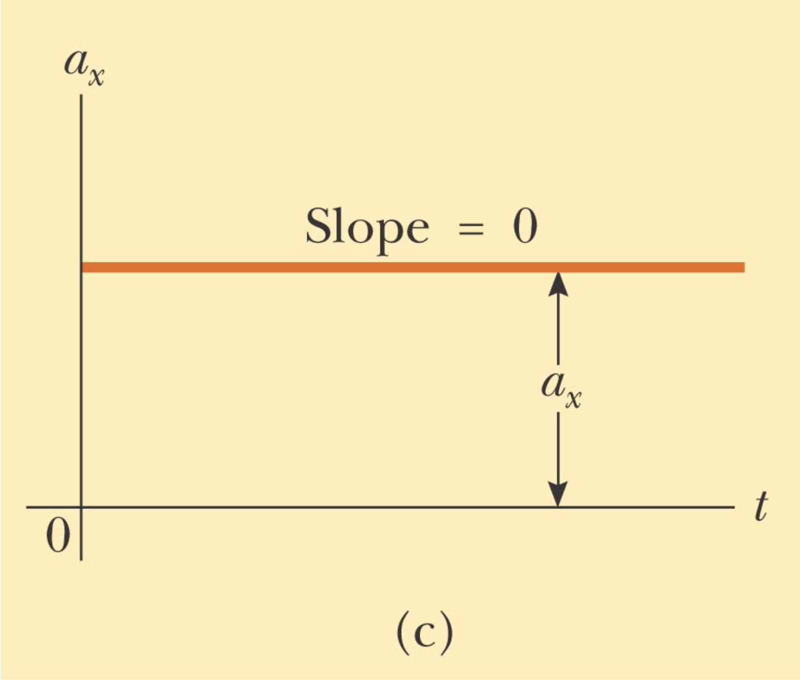
Graphical Look at Motion – velocity – time curve

- The slope gives the acceleration
- The straight line indicates a constant acceleration



Graphical Look at Motion – acceleration – time curve

- The zero slope indicates a constant acceleration



2.7 Freely Falling Objects

- A ***freely falling object*** is any object moving freely under the influence of gravity alone.
- It does not depend upon the initial motion of the object
  - Dropped – released from rest
  - Thrown downward
  - Thrown upward

Acceleration of Freely Falling Object

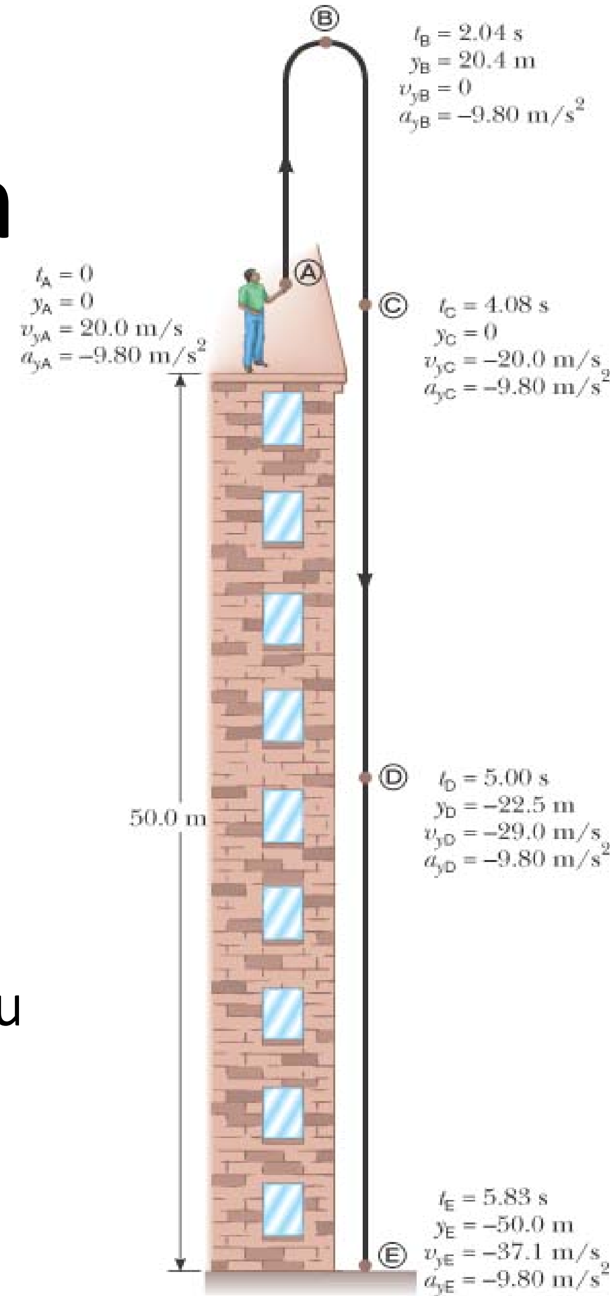
- The acceleration of an object in free fall is directed downward, regardless of the initial motion
- The magnitude of free fall acceleration is g = 9.80 m/s^2
  - g decreases with increasing altitude
  - g varies with latitude
  - 9.80 m/s^2 is the average at the Earth's surface

Acceleration of Free Fall, cont.

- We will neglect air resistance
- Free fall motion is constantly accelerated motion in one dimension
- Let upward be positive
- Use the kinematic equations with
- $a_y = g = -9.80 \text{ m/s}^2$

Free Fall Exam

- Initial velocity at A is upward (+) and acceleration is *g* (-9.8 m/s^2)
- At B, the velocity is 0 and the acceleration is *g* (-9.8 m/s^2)
- At C, the velocity has the same magnitude as at A, but is in the opposite direction
- The displacement is –50.0 m (it ends u 50.0 m below its starting point)



Kinematic Equations – General Calculus Form

$$a_x = \frac{dv_x}{dt}$$

$$v_x = \frac{dx}{dt}$$

Problem Solving – Conceptualize

- Think about and understand the situation
- Make a quick drawing of the situation
- Gather the numerical information
  - Include algebraic meanings of phrases
- Focus on the expected result
  - Think about units
- Think about what a reasonable answer should be

Problem Solving – Analyze

- Select the relevant equation(s) to apply
- Solve for the unknown variable
- Substitute appropriate numbers
- Calculate the results
  - Include units

- Problem (1)

- The position of a pinewood derby car was observed at various times; the results are summarized in the following table. Find the average velocity of the car for (a) the first second, (b) the last 3 s, and (c) the entire period of observation.

t (s)	0	1.0	2.0	3.0	4.0	5.0
x (m)	0	2.3	9.2	20.7	36.8	57.5

- Solution:

(a) $\bar{v} = \boxed{2.30\ \text{m/s}}$

(b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5\ \text{m} - 9.20\ \text{m}}{3.00\ \text{s}} = \boxed{16.1\ \text{m/s}}$

(c) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5\ \text{m} - 0\ \text{m}}{5.00\ \text{s}} = \boxed{11.5\ \text{m/s}}$

- **Problem (2)**
- A particle moves along the x axis according to the equation
- $x = 2.00 + 3.00\,t - 1.00\,t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.

- **Solution:**

$$x = 2.00 + 3.00t - t^2,\quad v = \frac{dx}{dt} = 3.00 - 2.00t,\quad a = \frac{dv}{dt} = -2.00$$

At $t = 3.00$ s:

(a) $x = (2.00 + 9.00 - 9.00)\ \text{m} = \boxed{2.00\ \text{m}}$

(b) $v = (3.00 - 6.00)\ \text{m/s} = \boxed{-3.00\ \text{m/s}}$

(c) $a = \boxed{-2.00\ \text{m/s}^2}$

- Problem (3)
- A jet plane lands with a speed of
- 100 m/s and can accelerate at a maximum rate of –5.00 m/s^2 as it comes to rest. (a) From the instant the plane touches the runway, what is the minimum time interval needed before it can come to rest? (b) Can this plane land on a small tropical island airport where the runway is 0.800 km long?
- Solution:

(a) $v_i = 100\text{ m/s}$, $a = -5.00\text{ m/s}^2$, $v_f = v_i + at$ so $0 = 100 - 5t$, $v_f^2 = v_i^2 + 2a(x_f - x_i)$ so $0 = (100)^2 - 2(5.00)(x_f - 0)$. Thus $x_f = 1000\text{ m}$ and $t = \boxed{20.0\text{ s}}$.

(b) At this acceleration the plane would overshoot the runway: $\boxed{\text{No}}$.

- Problem (4)
- For many years Colonel John P. Stapp, USAF, held the world's land speed record. On March 19, 1954, he rode a rocket-propelled sled that moved down a track at 632 mi/h. He and the sled were safely brought to rest in 1.40 s (Fig. P2.31). Determine (a) the negative acceleration he experienced and (b) the distance he traveled during this negative acceleration.
- Solution:

(a) $$a = \frac{v_f - v_i}{t} = \frac{632\left(\frac{5\,280}{3\,600}\right)}{1.40} = \boxed{-662\ \text{ft/s}^2} = -202\ \text{m/s}^2$$

(b) $$x_f = v_i t + \frac{1}{2}at^2 = (632)\left(\frac{5\,280}{3\,600}\right)(1.40) - \frac{1}{2}(662)(1.40)^2 = \boxed{649\ \text{ft}} = \boxed{198\ \text{m}}$$

- Problem (5)
- A golf ball is released from rest from the top of a very tall building. Neglecting air resistance, calculate the position and velocity of the ball after 1.00, 2.00, and 3.00 s.
- Solution:

$y_i = 0$, $v_i = 0$, and $a = -g = -9.80\ \text{m/s}^2$.

$$y_f - y_i = v_i t + \frac{1}{2} a t^2 \Rightarrow y_f = -\frac{1}{2} g t^2 = -\frac{1}{2}\left(9.80\ \text{m/s}^2\right) t^2$$

$$v_f = v_i + at \Rightarrow v_f = -gt = -\left(9.80\ \text{m/s}^2\right) t.$$

(a)

at $t = 1.00$ s: $y_f = -\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(1.00\ \text{s})^2 = \boxed{-4.90\ \text{m}}$

at $t = 2.00$ s: $y_f = -\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(2.00\ \text{s})^2 = \boxed{-19.6\ \text{m}}$

at $t = 3.00$ s: $y_f = -\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(3.00\ \text{s})^2 = \boxed{-44.1\ \text{m}}$

(b)

at $t = 1.00$ s: $v_f = -\left(9.80\ \text{m/s}^2\right)(1.00\ \text{s}) = \boxed{-9.80\ \text{m/s}}$

at $t = 2.00$ s: $v_f = -\left(9.80\ \text{m/s}^2\right)(2.00\ \text{s}) = \boxed{-19.6\ \text{m/s}}$

at $t = 3.00$ s: $v_f = -\left(9.80\ \text{m/s}^2\right)(3.00\ \text{s}) = \boxed{-29.4\ \text{m/s}}$

- Problem (6)
- A student throws a set of keys vertically upward to her sorority sister, who is in a window 4.00 m above. The keys are caught 1.50 s later by the sister's outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?
- Solution:

(a) $y_f - y_i = v_i t + \frac{1}{2} a t^2 : 4.00 = (1.50) v_i - (4.90)(1.50)^2$

and $v_i = \boxed{10.0 \text{ m/s upward}}$.

(b) $v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68 \text{ m/s}$

$v_f = \boxed{4.68 \text{ m/s downward}}$

- Problem (7)
- It is possible to shoot an arrow at a speed as high as 100 m/s. (a) If friction is neglected, how high would an arrow launched at this speed rise if shot straight up? (b) How long would the arrow be in the air?
- Solution:

(a) Consider the upward flight of the arrow.

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$0 = (100\ \text{m/s})^2 + 2(-9.8\ \text{m/s}^2)\Delta y$$

$$\Delta y = \frac{10\,000\ \text{m}^2/\text{s}^2}{19.6\ \text{m/s}^2} = \boxed{510\ \text{m}}$$

(b) Consider the whole flight of the arrow.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 0 + (100\ \text{m/s})t + \frac{1}{2}(-9.8\ \text{m/s}^2)t^2$$

The time of flight is given by

$$t = \frac{100\ \text{m/s}}{4.9\ \text{m/s}^2} = \boxed{20.4\ \text{s}}.$$