Consider the following linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 5 & 0 & -1 \\ -1 & 3 & 0 \\ 0 & -1 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}.$$

Find the matrix form of the Gauss-Seidel iterative method and then compute the number of iterations needed to get an accuracy within 10^{-4}, using Gauss-Seidel iterative method and $\mathbf{x}^{(0)} = [0.5, 0.5, 0.5]^T$.

Question 2: Consider the following linear system of equations [6 Marks]

$$\begin{aligned} 2x_1 + x_2 &= 3 \\ x_1 + 8x_2 + x_3 &= 10 \\ x_2 + 2x_3 &= 3 \end{aligned}$$

If $\mathbf{x} = [1, 1, 1]^T$ be the exact solution of the system, then using Jacobi iterative method and $\mathbf{x}^{(0)} = [0.5, 0.5, 0.5]^T$, compute the absolute error $\|\mathbf{x} - \mathbf{x}^{(2)}\|$. How many iterations needed to get an accuracy within 10^{-4} using Jacobi iterative method.

Consider the following linear system:

$$\begin{aligned} x_1 - x_2 + x_3 &= -2 \\ 2x_1 + x_2 - 2x_3 &= 6 \\ 2x_1 - x_2 - x_3 &= 1 \end{aligned}$$

If $\mathbf{x}^* = [1.01, 2.01, -0.98]^T$ is an approximate solution of the given linear system, then find the corresponding residual vector $\mathbf{r}$ and estimate the relative error $\dfrac{\|\mathbf{x} - \mathbf{x}^*\|}{\|\mathbf{x}\|}$.