King Saud University: Mathematics Department Math-254
First Semester 1445 H Final Examination
Maximum Marks = 40 Time: 180 mins.

Name of the Student:———————————— **I.D. No.** ————————-

Name of the Teacher:———————————— **Section No.** ————————

Note: Check the total number of pages are Six (6).

(**15 Multiple choice questions and Two (2) Full questions**)

<u>**The Answer Tables for Q.1 to Q.15 :**</u> Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Q. No.	11	12	13	14	15
a,b,c,d					

Quest. No.	Marks Obtained	Marks for Questions
Q. 1 to Q. 15		30
Q. 16		5
Q. 17		5
Total		40

Question 1: If $x_{n+1} = \dfrac{a}{b - \cos(x_n)}$, $n \geq 0$, is the fixed-point iterative form of the nonlinear equation $\dfrac{2}{x} + \cos(x) - 3 = 0$, then the value of the constants a and b are:

(a) $a = 2, b = 1$ (b) $a = 3, b = 2$ (c) $a = 2, b = 3$ (d) None of these

Question 2: The next iterative value of the root of $x^3 = 3x - 2$ using the secant method, if the initial guesses are -2.6 and -2.4 is:

(a) -2.2066 (b) -2.1066 (c) -2.3066 (d) None of these

Question 3: If the iterative scheme $x_{n+1} = x_n - k\dfrac{f(x_n)}{f'(x_n)}$, $n \geq 0$, converges at least quadratic to a simple root α, than the value of k is:

(a) k=1 (b) k=2 (c) k=3 (d) None of these

Question 4: The l_∞−norm of the inverse of the Jacobian matrix for the nonlinear system $x^2 + y^2 = 4, \;\; 2x - y^2 = 0$ using $[x_0, y_0]^t = [1, 1]^t$ is:

(a) 2 (b) 0.5 (c) 4 (d) None of these

Question 5: Let $A = \begin{bmatrix} 1.001 & 1.5 \\ 2 & 3 \end{bmatrix}$, then the determinant of a lower-triangular matrix L of the LU factorization using Crouts method is:

(a) 0.300 (b) 1.001 (c) 0.003 (d) None of these

Question 6: The l_∞−norm of the Jacobi iteration matrix of the following linear system $4x_1 - x_2 + x_3 = 7, \;\; 4x_1 - 8x_2 + x_3 = -21, \;\; -2x_1 + x_2 + 5x_3 = 15$ is:

(a) 0.625 (b) 0.5 (c) 0.4 (d) None of these

Question 7: Using Gauss-Seidel method and starting with $\mathbf{x}^{(0)} = [1.200, 0.467, 1.033]^t$, then the first approximation of the solution for the following linear system is:
$5x_1 + 2x_2 - x_3 = 6, \;\; x_1 + 6x_2 - 3x_3 = 4, \;\; 2x_1 + x_2 + 4x_3 = 7$ is:

(a) $\mathbf{x}^{(1)} = \begin{pmatrix} 1.024 \\ 1.006 \\ 0.987 \end{pmatrix}$ (b) $\mathbf{x}^{(1)} = \begin{pmatrix} 0.897 \\ 0.950 \\ 1.019 \end{pmatrix}$ (c) $\mathbf{x}^{(1)} = \begin{pmatrix} 1.220 \\ 0.980 \\ 0.895 \end{pmatrix}$ (d) None of these

Question 8: Let $A = \begin{bmatrix} 0 & \alpha \\ 1 & 1 \end{bmatrix}$ and $1 < \alpha < 2$. If the condition number $k(A)$ of the matrix A is 6, then α equals to

(a) 0.8 (b) 0.5 (c) 0.2 (d) None of these

Question 9: Let $x_0 = 2$, $x_1 = 2.5$, $x_2 = 4$ and $x_3 = 5.5$. If the best approximation of $f(x) = \dfrac{1}{x}$ at $x = 3$ using quadratic interpolation formula is $P_2(3) = 0.325$, then the value of the unknown point η in the error formula is equal to :

(a) 2.9201 **(b)** 2.7859 **(c)** 3.1472 **(d)** None of these

Question 10: If $x_0 = 0$, $x_1 = 1$, $x_2 = 3$ and for a function $f(x)$, the divided differences are $f[x_1] = 2$, $f[x_2] = 3$, $f[x_0, x_1] = 1$, $f[x_1, x_2] = \frac{1}{2}$, $f[x_0, x_1, x_2] = -\frac{1}{6}$. Then the approximation of $f(\frac{1}{2})$ using quadratic interpolation Newton formula is:

(a) 1.5417 **(b)** 4.1232 **(c)** 2.3481 **(d)** None of these

Question 11: Let $f(x) = x^3$ and $h = 0.1$. The absolute error for the approximation of $f'(0.2)$ using 2-point forward difference formula is:

(a) 0.0722 **(b)** 0.0711 **(c)** 0.0700 **(d)** None of these

Question 12: The absolute error for the approximation of the integral $\displaystyle\int_1^2 \frac{1}{x+1}\, dx$ using simple Trapezoidal's rule is:

(a) 0.1120 (b) 0.0112 (c) 0.0012 (d) None of these

Question 13: The approximation to the integral $\displaystyle\int_0^2 e^x\, dx$ using simple Simpson's rule is:

(a) 8.4207 (b) 7.4207 (c) 6.4207 (d) None of these

Question 14: For the initial value problem, $(x+1)y' + y^2 = 0, y(0) = 1, n = 1$, if the actual solution of the differential equation is $y(x) = \dfrac{1}{(1 + \ln(x+1))}$, then the absolute error by using Euler's method for the approximation of $y(0.05)$ is:

(a) 0.0350 **(b)** 0.0035 **(c)** 0.0042 **(d)** None of these

Question 15: Using the Taylor's method of order 2 to find the approximate value of $y(0.1)$ for the initial-value problem, $y' = e^{-2x} - 2y,\ y(0) = 0.1, n = 1$, is:

(a) 0.1620 **(b)** 0.1983 **(c)** 0.1846 **(d)** None of these

Question 16: Let $f(x) = \dfrac{3^x}{x}$ and $h = 0.1$ Compute the approximate value of $f''(3)$ and the absolute error. If $\max|f^{(4)}| = 6.1022$, then find the number of subintervals required to obtain the approximate value of $f''(3)$ within the accuracy 10^{-4}.

Question 17: Determine the number of subintervals required to approximate the integral $\int_0^2 \frac{1}{x+4}dx$, with an error less than 10^{-4} using composite Simpson's rule. Then approximate the given integral and compute the absolute error.