King Saud University: Mathematics Department Math-254
Second Semester 1446 H Second Midterm Exam.
Maximum Marks = 25 Time: 90 mins.

Questions: (6 + 6 + 7 + 6) Marks

Question 1:

Use Gauss elimination by partial pivoting to find the inverse A^{-1} involving α ($\neq 2$) of the following matrix,

$$A = \begin{pmatrix} 1 & \alpha & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}.$$

Then use A^{-1} with $\alpha = 3$ to solve the given linear system $A\mathbf{x} = [2, 2, 3]^T$.

Question 2:

Use LU decomposition by Dollittle's method to find the value(s) of $\alpha(\neq 2)$ for which the following matrix

$$A = \begin{pmatrix} 2 & \alpha & -1 \\ \alpha & 2 & 1 \\ -1 & 1 & 1 \end{pmatrix},$$

is singular. Compute the unique solution of the linear system $A\mathbf{x} = [1, 0, -1/2]^T$ by using the largest negative integer value of α.

Question 3:

Consider the following linear system

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & & & = & 3 \\ x_1 & + & 3x_2 & + & x_3 & = & 5 \\ & - & 2x_2 & + & 3x_3 & = & 1 \end{array}$$

Find the Gauss-Seidel iteration matrix T_G and show that $\|T_G\|_\infty > 1$. Use Gauss-Seidel method to compute the approximate solution of the system within accuracy 5×10^{-2}, using the initial solution $\mathbf{x}^{(0)} = [0.9, 0.9, 0.9]^T$. If $\mathbf{x} = [1, 1, 1]^T$ is the exact solution of the linear system, then compute the absolute error.

Question 4:

Consider a nonsingular linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 4 & 0 \\ 1 & 2 & 1 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} 4/3 & 1 & -8/3 \\ -1/3 & 0 & 2/3 \\ -2/3 & -1 & 7/3 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

If $\mathbf{b}$ of the given linear system is changed to $\mathbf{b}^* = [1, 1, 1.99]^T$, then how large a relative error can this change produce in the solution to the given linear system $A\mathbf{x} = \mathbf{b}$?