King Saud University: Mathematics Department Math-254
Second Semester 1446 H First Midterm Exam.
Maximum Marks = 25 Time: 90 mins.

Questions: (6 + 6 + 6 + 7) Marks

Question 1:

One of the possible rearrangement of the nonlinear equation $e^x = x + 2$, which has root in $[1, 2]$ is

$$x_{n+1} = g(x_n) = \ln(x_n + 2); \qquad n = 0, 1, \ldots.$$

Show that $g(x)$ has a unique fixed-point in $[1, 2]$. If $x_0 = 1.5$, then estimate the number of iterations n within accuracy 10^{-5}.

Question 2:

Successive approximations x_n to the desired root are generated by the scheme

$$x_{n+1} = \frac{e^{x_n}(x_n + 1) + 2x_n^2}{e^{x_n} + 3x_n}, \qquad n \geq 0.$$

Find the nonlinear equation $f(x) = 0$ and then show that it has a root in $[-1, 0]$. Use the Newton's method to find the second approximation of the root $\alpha = -0.7035$, starting with $x_0 = -0.5$. Compute the relative error.

Question 3:

Show that the order of convergence of the iterative scheme

$$x_{n+1} = \frac{x_n(x_n^2 + 3)}{3x_n^2 + 1}, \qquad n \geq 0,$$

at the fixed-point $\alpha = 1$ is at least cubically.

Question 4:

Show that the best iterative formula for computing the approximation of the root $\alpha = 0$ of the equation $x^3 e^{2x} = 0$ is

$$x_{n+1} = x_n - \frac{3x_n}{(3 + 2x_n)}, \qquad n \geq 0.$$

Use it to find the absolute error $|\alpha - x_2|$ using $x_0 = 0.1$. Show that the given iterative formula is converges quadratically to a root $\alpha = 0$.