King Saud University: Mathematics Department MAth-254
Second Semester 1446-47 H Final Examination
Maximum Marks = 40 Time: 180 mins.

Name of the Student:——————————— **I.D. No.** ——————————-

Name of the Teacher:——————————— **Section No.** ——————————

Note: Check the total number of pages are Six (6).

(**15 Multiple choice questions and Two (2) Full questions**)

The Answer Tables for Q.1 to Q.15 : Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {A, B, C or D} for the correct answer in the following box.

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A,B,C,D															

Quest. No.	Marks Obtained	Marks for Questions
Q. 1 to Q. 15		30
Q. 16		5
Q. 17		5
Total		40

Question 1: The number of bisections required to solve the equation $x^3 - x^2 = 1$ in $[1, 2]$ accurate to within 10^{-6} is:

(A) 20 (B) 11 (C) 15 (d) None of these

Question 2: When using Newton's method with $x_0 = 1$ for solving the equation $\cos(\pi x - \pi) - x = 0$, the first approximation x_1 is:

(A) 1.35 **(B)** 1.55 **(C)** 1.00 **(D)** None of These

Question 3: Which of the following sequences will converge faster to $\sqrt{5}$:

(A) $x_{n+1} = \frac{1}{3}[3x_n + 1 - \frac{x_n^2}{5}]$ **(B)** $x_{n+1} = x_n + 1 - \frac{x_n^2}{5}$ **(C)** $x_{n+1} = \frac{5}{x_n}$ **(D)** None of These

Question 4: The rate of convergence the iterative scheme $x_{n+1} = \frac{1}{2}(x_n^2 + 1) - \ln x_n$, $n \geq 0$ to $\alpha = 1$ is:

(A) Order 1 **(B)** Order 3 **(C)** Order 2 **(D)** None of These

Questions (5 - 7) are concerned with Linear System $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 0.5 \\ -2 & 1 \end{bmatrix}, \qquad A^{-1} = \begin{bmatrix} 0.5 & -0.25 \\ 1 & 0.5 \end{bmatrix}, \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}.$$

Question 5: In the LU factorization with $u_{ii} = 1,\ i = 1, 2$ of the matrix A, the matrix L is given by:

(A) $\begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix}$ **(D)** None of These

Question 6: For the given linear system, the l_∞-norm of the Jacobi iteration matrix T_J is equal to:

(A) 0.5 **(B)** 2.0 **(C)** 1.0 **(D)** None of these

Question 7: The relative error $\frac{\|\mathbf{x} - \mathbf{x}^*\|_\infty}{\|\mathbf{x}\|_\infty}$, where $x^* = [2, 0]^T$ be the approximation of the given linear system, is bounded by:

(A) 4.75 **(B)** 5.75 **(C)** 6.75 **(D)** None of These

Question 8: If $f(x) = x^2$ and the second order divided difference $f[\alpha, 2, 2] = 3$. Then the value of α is:

(A) 2 **(B)** 3 **(C)** 1 **(D)** None of These

Question 9: Using the Newton's interpolating polynomial of degree 2 for $f(x) = x \ln x + e^{-x}$ on the interval $[2, 4]$ with the points $x_0 = 2.0$, $x_1 = 3.0$, $x_2 = 4.0$, an error bound for the approximation of $f(2.4226)$ is:

(A) 0.00247 **(B)** 0.0247 **(C)** 0.0427 **(D)** None of These

Question 10: When using simple trapezoidal rule for approximating the integral $\int_1^{1.5} \frac{1}{x}\, dx$, we have the computed approximation:

(A) $\frac{5}{12}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{5}{14}$ **(D)** None of These

Question 11: If $f(0) = 3, f(1) = \frac{\alpha}{2}, f(2) = \alpha$, and Simpson's rule for $\int_0^2 f(x)\, dx$ gives 2, then the value of α is:

(A) -1.0 **(B)** 2.0 **(C)** 1.0 **(D)** None of These

Question 12: Using data points: $(0, 1), (0.1, 1.1), (0.2, 1.3), (0.3, 1.4), (0.4, 1.5), (0.5, 1.7)$, then the **best** approximate value of $f'(0)$ using 3-point difference formula is:

(A) 0.55 **(B)** 0.75 **(C)** 0.50 **(D)** None of These

Question 13: Let $f(x) = x \ln x + x$ and $x = 0.9, 1.3, 1.9, 2.1, 2.5, 3.2$. Then the absolute error for the approximation of $f''(1.9)$ is:

(A) 0.0930 **(B)** 0.0093 **(C)** 0.9300 **(D)** None of These

Question 14: Given initial-value problem $\frac{1}{x}y' - y^2 = 0,\ y(1) = 1$ and its exact solution is $y(x) = 2/(3 - x^2)$, then absolute error for the approximate value of $y(1.4)$ using Euler's method with $n = 2$ is:

(A) 0.3775 **(B)** 0.5377 **(C)** 0.3577 **(D)** None of These

Question 15: Given initial-value problem $e^y y' - e^x = 0,\ y(0) = 1$, the approximate value of $y(1)$ using Taylor's method of order 2 with $n = 2$ is:

(A) 1.4893 **(B)** 1.4983 **(C)** 1.4993 **(D)** None of These

Question 16: If $f(x) = x^2 + \cos 2x$ (x is in radian) and x-values are $\{-0.5, 0.0, 0.3, 0.5, 0.6, 1.0\}$. Use the quadratic Lagrange interpolating polynomial for equally spaced data points to find the best approximation of $0.16 + \cos 0.8$. Compute an error bound and the absolute error.

Question 17: Find the approximation of $\int_0^{1.25} f(x)\,dx$ by using the following set of data points using the best integration rule:

x	0.0	0.1	0.25	0.31	0.4	0.5	0.6	0.75	0.8	0.91	1.0	1.1	1.25
$f(x)$	0	0.1098	0.3099	0.4012	0.5494	0.7294	0.9246	1.2441	1.3574	1.6176	1.8415	2.1012	2.5115

The function tabulated is $f(x) = x^2 + \sin x$ (x is in radian), compute the absolute error and the number of subintervals approximate the given integral to within accuracy of 10^{-3} ?