King Saud University: Mathematics Department Math-254
Second Semester 1445 H Second Midterm Exam.
Maximum Marks = 25 Time: 90 mins.

Question 1: Use the Gaussian elimination with partial pivoting to find the solution and the determinant of the matrix A of the following linear system [6 Marks]

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & = & 0.5 \\ 2x_1 & - & 3x_2 & + & x_3 & = & -1 \\ -x_1 & - & 1.5x_2 & + & 2.5x_3 & = & -1 \end{array}$$

Question 2: Consider the following linear system $A\mathbf{x} = \mathbf{b}$, where [7 Marks]

$$A = \begin{pmatrix} \alpha & 4 & 1 \\ 2\alpha & -1 & 2 \\ 1 & 3 & \alpha \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 6 \\ 3 \\ 5 \end{pmatrix}.$$

Use LU decomposition by Dollittle's method to find the solution of the consistent system (Unique and infinitely many solutions).

Question 3: Consider the following linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 5 & 0 & -1 \\ -1 & 3 & 0 \\ 0 & -1 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}.$$

Find the matrix form of Gauss-Seidel iterative method and compute number of iterations needed to get an accuracy within 10^{-4}, using Gauss-Seidel iterative method and $\mathbf{x}^{(0)} = [0.5, 0.5, 0.5]^T$.

Question 4: Consider the following linear system:

$$\begin{array}{rcrcrcr} x_1 & - & x_2 & + & x_3 & = & -2 \\ 2x_1 & + & x_2 & - & 2x_3 & = & 6 \\ 2x_1 & - & x_2 & - & x_3 & = & 1 \end{array}$$

If $\hat{\mathbf{x}} = [1.01, 2.01, -0.98]^T$ is an approximate solution of the given system, then find the corresponding residual vector $\mathbf{r}$ and estimate the relative error.