

Question 1:

[6 Marks]

Show that the x-value of the intersection point (x, y) of the graphs $y = x^3 + 2x - 1$ and $y = \sin x$ is lying in the interval $[0.5, 1]$. Then use the secant method to find its second approximation, when $x_0 = 0.5$ and $x_1 = 0.55$. Also, find the intersection point.

Question 2:

[6 Marks]

Find a value of constant λ ($\neq -1$) to ensure the rapid convergence to the root 1.4650 of the iterative scheme

$$x_{n+1} = \frac{\lambda x_n + x_n^{-2} + 1}{\lambda + 1}, \quad n \geq 0.$$

Use Newton's method to find the absolute error $|\alpha - x_2|$, when $x_0 = 1.5$.

Question 3:

[7 Marks]

Show that the equation $(1 - x)^2 e^{1-x} = 0$ has multiple root $\alpha = 1$ with order of multiplicity 2. Develop the Modified Newton's formula for computing the approximation of this root and use it to find the second approximation x_2 using $x_0 = 0.75$. Show that the developed formula converges only quadratically to the root.

Question 4:

[6 Marks]

Consider the nonlinear system

$$\begin{aligned} x^3 + \alpha y^2 &= 21 \\ x^2 + 2y + 2 &= 0 \end{aligned}$$

If the determinant of the Jacobian matrix J is 18 when the initial approximation is $(x_0, y_0)^T = (1, -1)^T$, then use Newton's method to find the first approximation $(x_1, y_1)^T$.