King Saud University: Mathematics Department MAth-254
Second Semester 1445 H Final Examination
Maximum Marks = 40 Time: 180 mins.

Name of the Student:————————————— **I.D. No.** ——————————

Name of the Teacher:————————————— **Section No.** ——————————

Note: Check the total number of pages are Six (6).

(**15 Multiple choice questions and Two (2) Full questions**)

The Answer Tables for Q.1 to Q.15 : Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a,b,c,d															

Quest. No.	Marks Obtained	Marks for Questions
Q. 1 to Q. 15		30
Q. 16		5
Q. 17		5
Total		40

Question 1: If $f(x) = xe^{-x}$ and $x_0 = 4$, then the first approximation x_1 by Newton's method is:

(a) 3.3333 **(b)** 5.3333 **(c)** 4.3333 **(d)** None of these

Question 2: Using the best iterative formula, the first approximation of the multiple root of the nonlinear equation $1 - \cos x = 0$, taking $x_0 = 0.1$ is:

(a) 0.0980 **(b)** 0.0890 **(c)** 0.0098 **(d)** None of these

Question 3: If the determinant of the Jacobian matrix of the system of nonlinear equations $x^2 + \alpha y^2 = 1,\ xy = 1$ at the point $(1, 1)$ is -4, then the value of α is:

(a) 2 **(b)** 3 **(c)** 1.5 **(d)** None of these

Question 4: If the matrix $A = \begin{pmatrix} 3 & 1 \\ 6 & 1 \end{pmatrix}$ is factored as LU using Doolliitle's method, where L is a lower triangular matrix, and U is an upper triangular matrix, then the solution of the system $L\mathbf{y} = [-1,\ 0]^t$ is :

(a) $[-1,\ 6]^t$ **(b)** $[-1,\ -2]^t$ **(c)** $[-1,\ 2]^t$ **(d)** None of these

Question 5: For the linear system $2x+y = 4,\ x+2y = 5$, then the l_∞-norm of the Gauss-Seidal iteration matrix T_G is equal to:

(a) 0.25 **(b)** 0.5 **(c)** 0.75 **(d)** None of these

Question 6: If $A = \begin{pmatrix} 1 & 1 \\ 1 & 0.5 \end{pmatrix}$ and l_∞-norm of the residual vector $\mathbf{r}$ is 0.005, then the error bound for the relative error in the solution of the linear system $A\mathbf{x} = [1, -0.5]^T$ is:

(a) 0.040 **(b)** 0.020 **(c)** 0.025 **(d)** None of these

Question 7: Using data points: $(0, f(0))$ and $(\pi, f(\pi))$. Then the approximation of the function $f(x) = 2\cos x$ at $\pi/2$ by a linear Lagrange polynomial is:

(a) $\pi/2$ **(b)** 0.0 **(c)** $\pi/4$ **(d)** None of these

Question 8: Using data points: $(0, 1), (1, 2), (2, 3)$, if $L_0(1.5) = -0.125$ and $L_2(1.5) = 0.375$, then the approximate value of $f(1.5)$ by a quadratic Lagrange polynomial is:

(a) 2.5 **(b)** 1.5 **(c)** 3.5 **(d)** None of these

Question 9: If $f(x) = \dfrac{3}{x}$ and $f[1,1,1,2] = \alpha$, then α is equal to:

(a) 1.5 **(b)** -1.5 **(c)** -4.5 **(d)** None of these

Question 10: Let $f(x) = x^3$ defined on $[0.2, 0.3]$. Then absolute error using the Two-point difference formula for the approximation of $f'(0.2)$ is:

(a) 0.12 **(b)** 0.07 **(c)** 0.19 **(d)** None of these

Question 11: If $f''(x) = x^4 f(x)$ and $f(0) = 1, f(0.5) = \alpha, f(0.7) = 1.75, f(1) = 2$. Then using 3-point central difference formula for $f''(x)$, the value of α is:

(a) 1.8484 **(b)** 1.9999 **(c)** 1.4884 **(d)** None of these

Question 12: If $f(1) = 0.5, f(1.5) = \alpha, f(1.8) = 1.5, f(2) = 2.5, f(2.5) = 3$ and using the best integration rule the value of $\int_1^{2.5} f(x)\, dx = 3$, then the value of α is:

(a) 1.75 **(b)** 1.5 **(c)** 2.25 **(d)** None of these

Question 13: If $\int_1^2 \dfrac{1}{x+1}\, dx = 0.4055$, then using simple Simpson's rule, the absolute error in the approximation is:

(a) 0.0112 **(b)** 0.0001 **(c)** 0.0025 **(d)** None of these

Question 14: For the initial value problem, $y' + x^2 = y + 1, y(0) = 0.5, n = 1$, if the actual solution of the differential equation is $y(x) = (x+1)^2 - 0.5e^x$, then the absolute error by using Euler's method for the approximation of $y(0.2)$ is:

(a) 0.0293 **(b)** 0.0392 **(c)** 0.0329 **(d)** None of these

Question 15: Given $4y' - y = 0,\ y(0) = 1$, the approximate value of $y(0.5)$ using Taylor's method of order two when $n = 1$ is:

(a) 1.1331 **(b)** 1.1328 **(c)** 1.2839 **(d)** None of these

Question 16: Let $f(x) = \ln(x+2)$ and $x_0 = 0, x_1 = 0, x_2 = 1, x_3 = 1$, find the best approximation of $\ln(2.5)$ by using the cubic Newton's polynomial. Compute absolute error and the error bound.

Question 17: The function tabulated below is $f(x) = x \ln x + x$.

x	0.9000	1.3000	1.5000	1.6000	1.9000	2.3000	2.5000	3.1000
$f(x)$	0.8052	1.6411	2.1082	2.3520	3.1195	4.2157	4.7907	6.6073

Find the approximation of $(\ln 1.9 + 2)$ using three-point formula for $f'(x)$ for smaller value of h. Compute the absolute error and the number of subintervals required to obtain the approximate value of $(\ln 1.9 + 2)$ within the accuracy 10^{-2}.