

Hyperbolic and Inverse Hyperbolic Trigonometric Functions

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- 2 Derivatives of Hyperbolic Trigonometric Functions
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$$\begin{aligned} \int \frac{\sinh(\sqrt{x})}{\sqrt{x}} dx &= \int \sinh(u) \cdot 2du = 2 \int \sinh(u) du \\ &= 2 \cosh(u) + c \end{aligned}$$

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Now we have $y = \sinh(\ln(2 \pm \sqrt{3})) = \pm\sqrt{3}$. The required points are $(\ln(2 \pm \sqrt{3}), 2 \pm \sqrt{3})$.