Grading scheme Final106

Q1)

a) $\int e^{-3x}\tan e^{-3x}\,dx = \frac{-1}{3}\int \frac{sinu}{cosu}du = \frac{1}{3}\ln|\cos(e^{-3x})| + C$

(2) + (1)

b) $\frac{1}{3}\int_1^4 3(x-1)^2dx = 9 = 3(c-1)^2$ so $c = 1+\sqrt{3}$

(as $1-\sqrt{3}$ is not in [1,4])

(2) + (1)

c) $\int \frac{dx}{x\sqrt{x^4-9}} = \frac{1}{2}\int \frac{du}{u\sqrt{u^2-9}} = \frac{1}{6}\sec^{-1}\left(\frac{x^2}{3}\right) + C$

(2) + (1)

Q2) a) $\int \frac{dx}{x\sqrt{1-x^5}} = \frac{2}{5}\int \frac{du}{u\sqrt{1-u^2}} = -\frac{2}{5}\operatorname{sech}^{-1}(\ x^{5/2}) + C$

(2) + (1)

b) $\int x^2coshxdx = x^2sinhx - 2\int xsinhxdx$

$= x^2sinhx - 2(xcoshx - sinhx) + C$

(1.5) + (1.5)

b) $\int (sinx)^5(cosx)^8dx = -\int(1-u^2)^2u^8du$

$$= -\frac{(cosx)^{13}}{13} + 2\frac{(cosx)^{11}}{11} - \frac{(cosx)^9}{9} + C$$

(1.5) +(1.5)

Q3) a) $y = (e^x + 3x)^{1/x}$

$$\lim_{x\to 0^+} lny = \lim_{x\to 0^+}\frac{e^x+3}{e^x+3x} = 4$$

So $\lim_{x\to 0^+} y = e^4$

(2,5) + (0,5)

b)

$$\int \frac{6x-11}{(x-1)^2}dx = \int \frac{6}{x-1} - \frac{5}{(x-1)^2}dx$$

$$= 6\ln|x-1| + \frac{5}{x-1} + C$$

(1) +(1)

c) $\int \frac{dx}{\sqrt{7-x^2+6x}} = \int \frac{dx}{\sqrt{16-(x-3)^2}} = \sin^{-1}\left(\frac{x-3}{4}\right) + C$

(1) +(1)

Q4

a) $\int_0^c \frac{cosxdx}{\sqrt{1-sinx}} = \int_0^{sinc} \frac{du}{\sqrt{1-u}} = [-2\sqrt{1-u}]_0^{sinc}$ **(2)**

$$\to 2 \ \ as\ c \to \pi/2$$

So the integral converges with a value of 2 **(1)**

b) Graph **(1)**

Intersections at $x = 0\ and\ x = -2$ **(0,5)**

$A = \int_{-2}^{0} 4 - x^2 - (2x+4)dx = \frac{4}{3}$ **(1.5)**

c) $V = 4\pi \int_0^1 xsinhxdx = 4\pi[xcoshx - sinhx]_0^1$

$= 4\pi(e^{-1})$ **(2) +(1)**

Q5

a) $L = \int_0^{\pi/2} \sqrt{x'(t)^2 + y'(t)^2}dt = \int_0^{\pi/2} \sqrt{2}e^t dt$

$$= \sqrt{2}(e^{\frac{\pi}{2}} - 1)$$

(2.5) +(0.5)

b) Graph **(1)**

Intersections points

$$2 - 2cos\theta = 2cos\theta$$

$\theta = \frac{\pi}{3}$ or $\theta = \frac{4\pi}{3}$ **(0.5)**

$$A = \int_{\pi/3}^{\pi} (2 - 2cos\theta)^2 - (2cos\theta)^2 d\theta$$

$$= [4\theta - 8sin\theta]_{\pi/3}^{\pi} = 8(\frac{\pi}{3} + \frac{\sqrt{3}}{2})$$ **(1.5)**