Grading schemeAP22

Q1)

a) $\lim_{x\to 0}\frac{x-sinx}{x^3}=\lim_{x\to 0}\frac{1-\cos x}{3x^2}=\lim_{x\to 0}\frac{sinx}{6x}=\frac{1}{6}$ **(1)+(1)+(1)**

b) $\int e^{-x}cos4xdx=\frac{1}{4}e^{-x}sin4x+\frac{1}{4}\int e^{-x}sin4xdx$

$=\frac{1}{4}e^{-x}sin4x+\frac{1}{4}(\frac{-e^{-x}cosx}{4}-\frac{1}{4}\int e^{-x}cos4xdx)$

Thus $\int e^{-x}cos4xdx=\frac{1}{17}e^{-x}(4sin4x-cos4x)+C$

(2)+(1)

c) $\int(tanx)^5(secx)^5dx=\int(u^2-1)^2u^4du$

$=\frac{(secx)^9}{9}-2\frac{(secx)^7}{7}+\frac{(secx)^5}{5}+C$ **(2)+(1)**

Q2) a) $\int sin4x.sin2xdx=\frac{1}{2}\int(cos2x-cos6x)dx$

$=\frac{1}{4}sin2x-\frac{1}{12}sin6x+C$ **(1)+(1)**

b) $\int\frac{dx}{x^2\sqrt{1+x^2}}=\int\frac{sec\theta.d\theta}{(tan\theta)^2}=\int\frac{cos\theta.d\theta}{(sin\theta)^2}=\frac{-1}{sin\theta}+c$

$=-\frac{\sqrt{1+x^2}}{x}+C$ **(2.5)+(0.5)**

c) $\frac{2x^2+3x+4}{(x+2)^2(x-1)}=\frac{1}{x-1}+\frac{1}{x+2}-\frac{2}{(x+2)^2}$

$\int\frac{2x^2+3x+4}{(x+2)^2(x-1)}dx=\ln|x-1|+\ln|x+2|+\frac{2}{x+2}+C$

(1.5) +(1.5)

Q3) a) $\int\frac{dx}{5+5cosx+4sinx}$

$$= \int \frac{du}{5+4u} = \frac{1}{4}\ln|5+4\tan(\frac{x}{2})| + C$$

(2)+(1)

b) $\int \frac{dx}{\sqrt{x^2-4x+5}} = \int \frac{du}{\sqrt{u^2+1}} = \sinh^{-1}(x-2) + C$

(1) +(1)

c) $\int_0^C \frac{x^2dx}{(1+x^3)^2} = \frac{1}{3}\int_0^{C^3} \frac{du}{(1+u)^2} = -\frac{1}{3}\left(\frac{1}{1+C^3} - 1\right)$ **(2)**

and $-\frac{1}{3}\left(\frac{1}{1+C^3} - 1\right) \to \frac{1}{3}\ as\ C \to \infty$

So the integral converges and its value is 1/3.

(1)