King Saud University

College of Science

Phys 105

Physics for Architecture Students

Mechanics

Chapter 1

Physics Quantities and Vectors

Scalar and vector quantities

A scalar quantity has only a magnitude, and is determined by a number and a unit, for example:

1. **Distance** : A car is driven for a distance of 2.3 km. A biker travels 7 m from the small tree to the big tree.
2. **Mass**: The mass of the human body is on average 70 Kg.
3. **Size**: The volume of the study hall is about 300 m^3.
4. **Frequency**: The frequency of the electrical current in homes for the 110 V line is 60 Hz.

- Similar scalar quantities are added and subtracted by ordinary mathematical methods.

Scalar and vector quantities

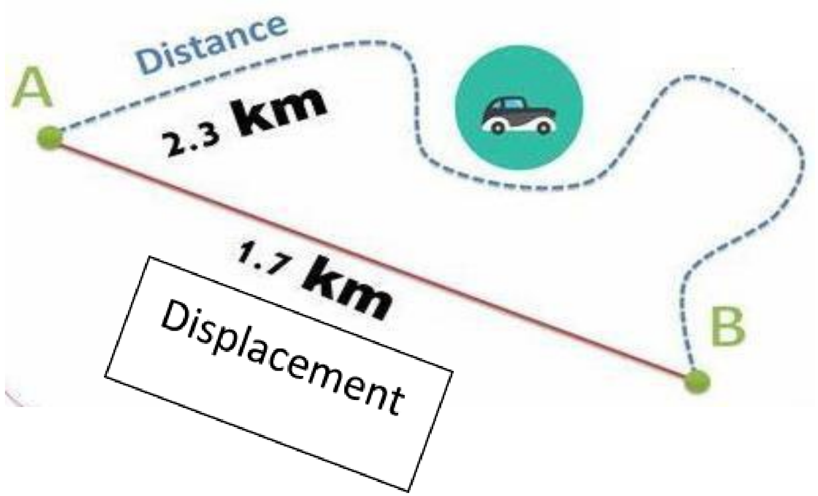
Vector quantities have both magnitude and direction, and they are determined by a number, a unit, and direction. For example:

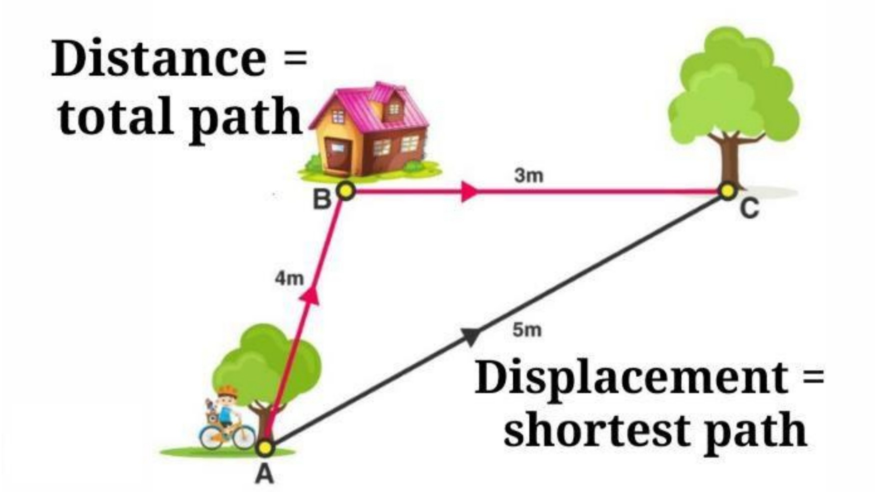
1. **Force**: A man applies a downward force of 10 N on a table.
2. **Velocity**: A train is traveling at a constant speed of 600 km/h towards the southwest.
3. **Displacement**: Displacement travelled by the car from A to B equals 1.7 km. Displacement travelled by the biker from Tree at A to Tree at C equals 5 m.

Distance versus displacement:

The distance of an object can be defined as the complete path traveled by the object. Displacement is the shortest distance between the initial and final position of an object.

Distance vs. Displacement





- Vector quantities are usually written by bold letters e.g. the force **F** and the velocity **v** or by placing an arrow sign on the quantity symbol ;therefore, we write $\vec{F}$, and $\vec{v}$
- When similar vector quantities are added and subtracted, the direction must be considered, and so it is necessary to know the methods of adding or subtracting vectors.

Vector Addition: the graphical method: Example 1

The vector is graphically represented by a straight line with an arrow at its end, so that the length of the line is proportional to the amount of the vector quantity and the direction of the arrow represents the direction of the vector quantity.

Example 1: A vector **A** has a different length and magnitude than the vector **B** .

We usually express the length or magnitude of a vector in the following way:

- The length (or magnitude) of the vector A = $|\boldsymbol{A}| = \vec{A}$
- The length (or magnitude) of the vector B = $|\boldsymbol{B}| = \overrightarrow{\mathrm{B}}$

Vector Addition: First: the graphical method

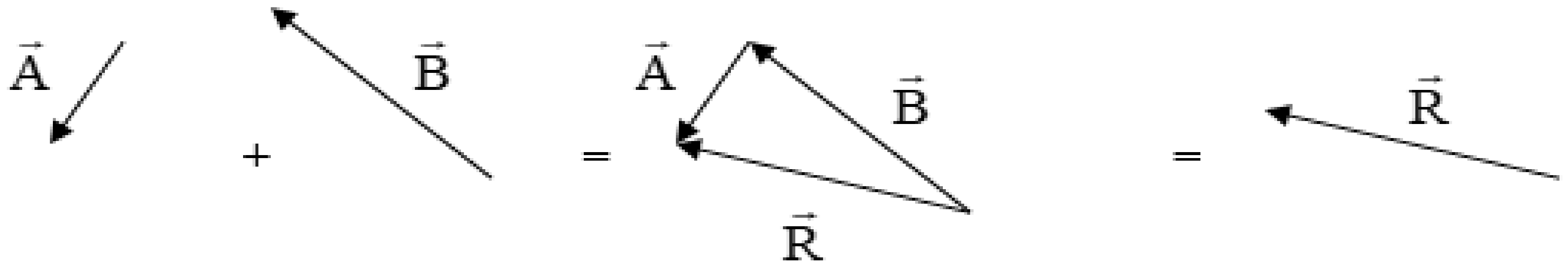
Addition of vectors A and B graphically is explained by the following :

- We draw **B** so that its beginning is at end of **A**, and then we connect a beginning of **A** and end of **B** by the vector **R**, which represents the vector summation of the two vectors, that is:

$$\vec{R} = \vec{A} + \vec{B}$$

- The same result can be obtained by changing the order, that is,

$$\vec{R} = \vec{B} + \vec{A}$$

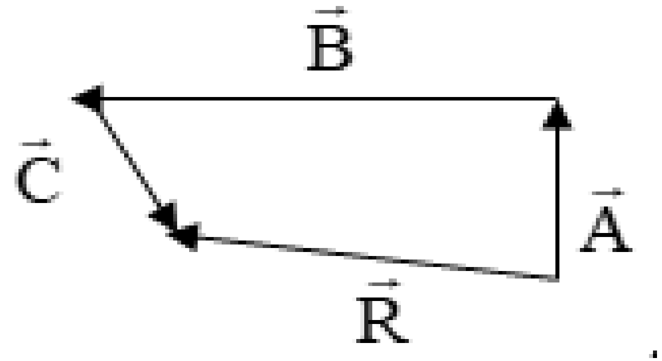


Vector Addition: Example 2

We follow the same method as before when adding more than one vector, as it is explained in the following example:

Example 2:

A car travels 30 km north, then moves west for 50 km and then goes southwest for 20 km. What is the distance between the start and end points?



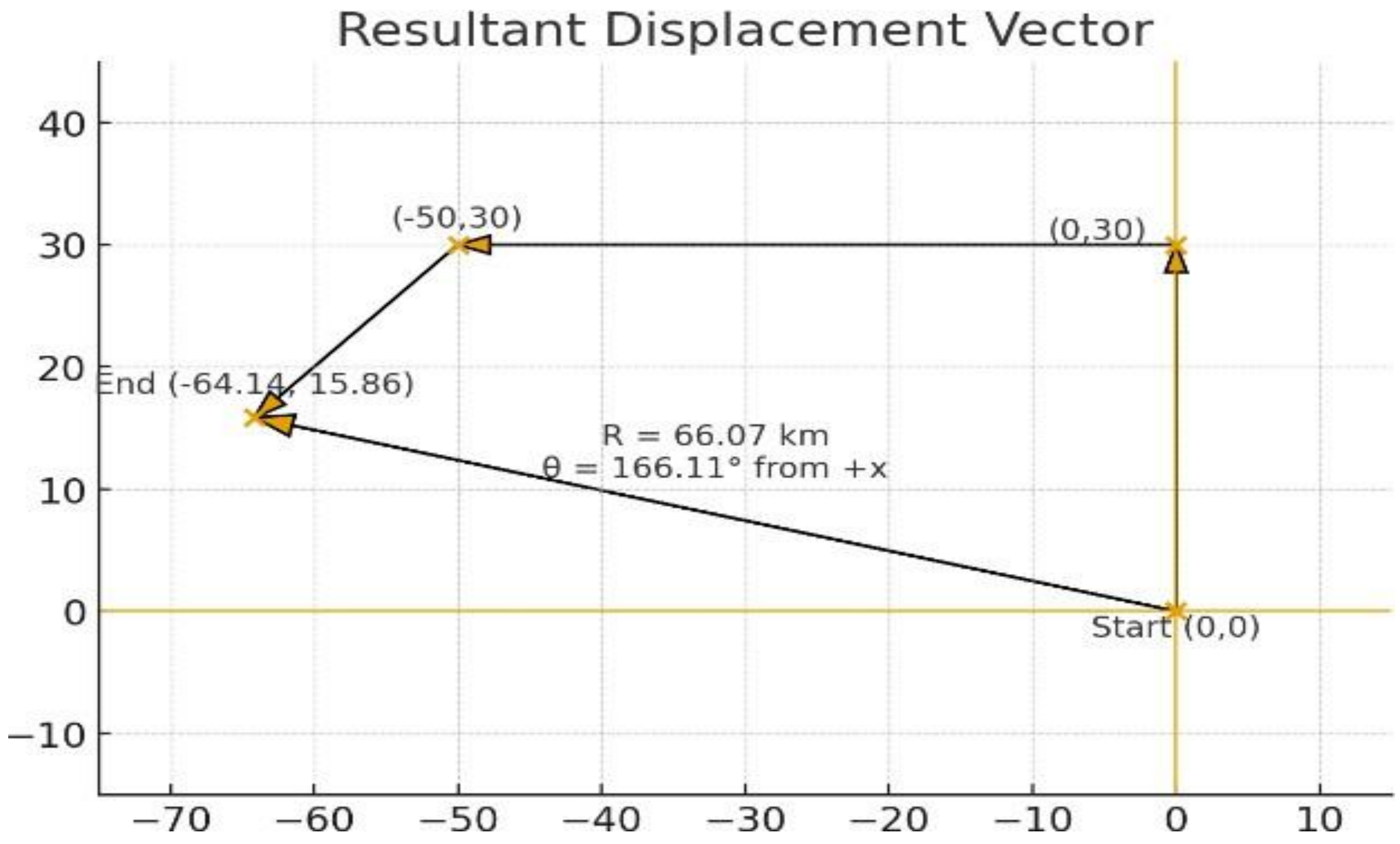
Solution:

- We call the northward displacement A. We call the westward displacement B. We call the southwestward displacement the vector C.
- The resultant is the vector summation of the three vectors, and we define them by connecting the beginning of the first vector with the end of the last vector and we write: $\vec{R} = \vec{A} + \vec{B} + \vec{C}$
- and the direction as shown in the figure is northwest, and its magnitude is:
- Resultant (**R**)= 48.8 km Direction: North-West (NW) at $\theta = 166^{o}$

Vector Addition: First: the graphical method

How did we make measurement:

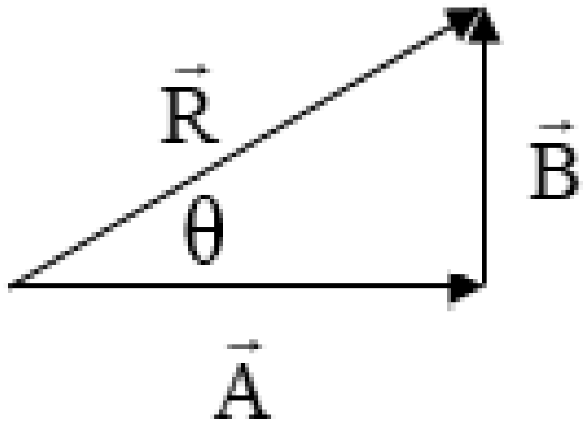
The magnitude (length) was determined by direct measurement after taking a suitable scale for the drawing.

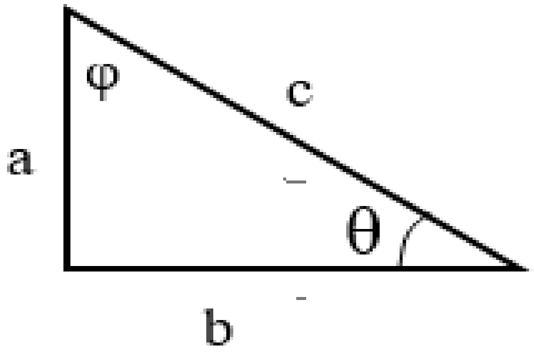


Vector Addition: Second: The trigonometric method

Although the magnitude and direction of the resultant R for two or more vectors can be determined by using graphical drawing, this method is not completely accurate. To get an accurate result of the resultant, we use triangles for the case of two vectors at right angles to one another(orthogonal vectors).

For a right triangle, we have:





$\sin \varphi = b/c$ $\sin \Theta = a/c$

$\cos \phi = a/c$ $\cos \Theta = b/c$

$\tan \phi = b/a$ $\tan \Theta = a/b$

Vector Addition: Second: The trigonometric method

$$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2}$$

$$a^2 = c^2 - b^2 \Rightarrow a = \sqrt{c^2 - b^2}$$

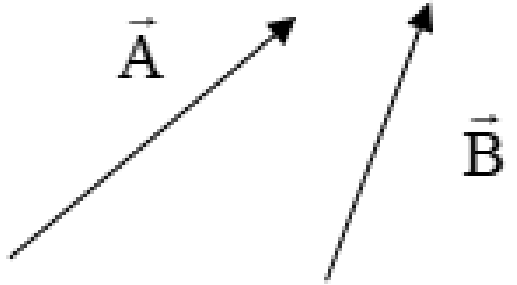
$$b^2 = c^2 - a^2 \Rightarrow b = \sqrt{c^2 - a^2}$$

- We also have the well-known relationship: the sum of the angles of a triangle =180
- So in a right-angled triangle, the sum of the two angles is: $\theta + \varphi = 90^o$
- It is appropriate to use this method in the following case:
- If we have two perpendicular vectors **A** and **B**, it is easy to find the resultant **R** by applying the Pythagorean theorem: $R = \sqrt{A^2 + B^2}$
- The direction of **R** can also be determined by knowing the angle θ from the relationship: $tan\theta = \frac{B}{A} \rightarrow \theta = tan^{-1}\frac{B}{A}$

But if the vectors **A** and **B** are not perpendicular, we use the component method to add the vectors.

Vector Addition: Third: Component method

- If we have two non-orthogonal vectors, then to get the vector addition for them, we first analyze each of them into two orthogonal components, one on the x direction and the other on the y direction, and we use the method of adding vectors graphically to get the following:



$$\vec{A} = \vec{A}_x + \vec{A}_y$$

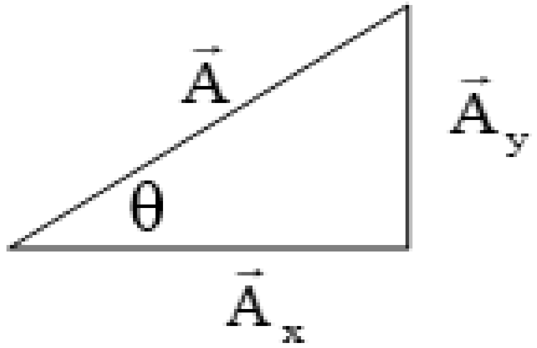
$$\vec{B} = \vec{B}_x + \vec{B}_y$$

- We use the trigonometric method to determine the magnitude of components for each vector as follows:

- In the right-angled triangle related to the vector A :

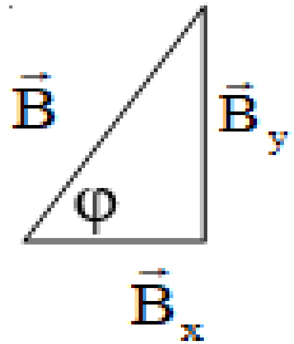
$$\sin\theta = \frac{A_y}{A} \rightarrow A_y = A\sin\theta$$

$$\cos\theta = \frac{A_x}{A} \rightarrow A_x = A\cos\theta$$



Vector Addition: Third: Component method

- In the same way for the right-angled triangle of the vector B :



$$sin\varphi = \frac{B_y}{B} \to B_y = Bsin\varphi$$

$$cos\varphi = \frac{B_x}{B} \to B_x = Bcos\varphi$$

- Then we add the components of the vectors for each direction separately, and we get the resultant in the x direction from the relationship:

$$\vec{R}_x = \vec{A}_x + \vec{B}_x$$

- The resultant in the y direction from of the relationship:

$$\vec{R}_y = \vec{A}_y + \vec{B}_y$$

Vector Addition: Third: Component method

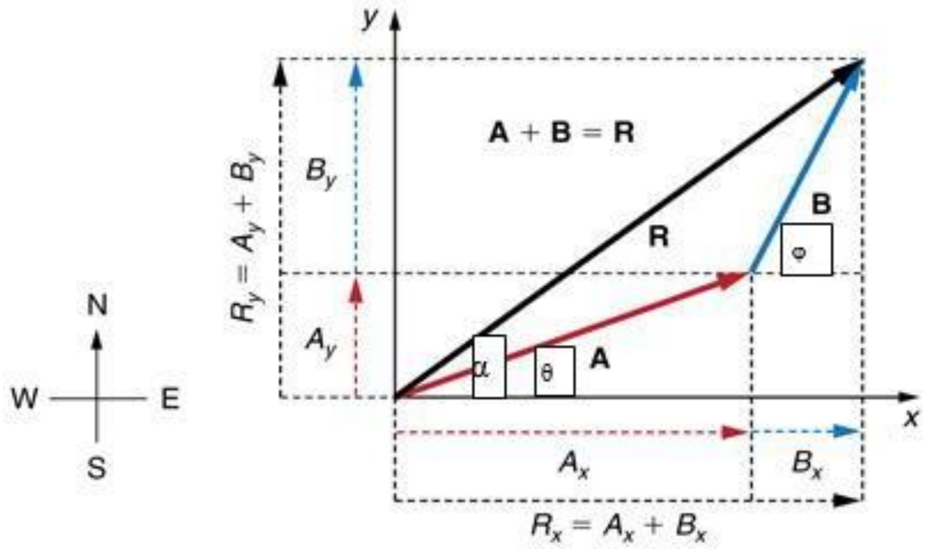
Since R_x and R_y are two perpendicular components, we apply the Pythagorean theorem to get the magnitude of the space vector, which represents the resultant R as follows:

$$R = \sqrt{R_x^2 + R_y^2}$$

We find the direction of R from the trigonometric relationship:

$$tan\alpha = \frac{R_y}{R_x}$$

$$\Longrightarrow \alpha = tan^{-1}\frac{R_y}{R_x}$$

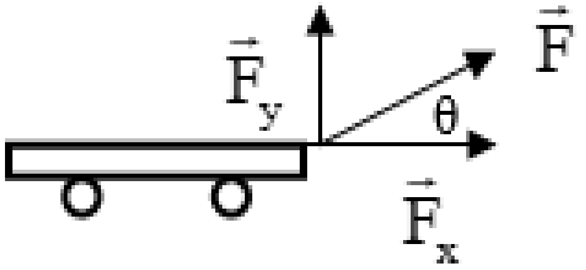


Vector Addition: Example 3

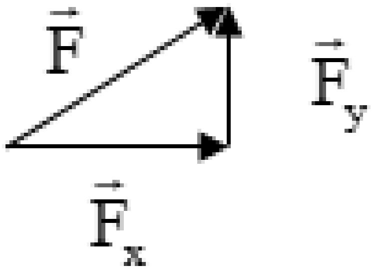
A man pulls the cart as shown in the figure with an inclined force **F** of 10 N such that $\theta = 30^o$. Determine the magnitude of the horizontal and vertical components of the force **F**.

solution:

Looking at the right-angled triangle, we find that:



$$sin\theta = \frac{F_y}{F}$$
$$\Longrightarrow F_y = Fsin\theta = 10sin30 = 5$$
$$cos\theta = \frac{F_x}{F}$$
$$\Longrightarrow F_x = Fcos\theta = 10cos30 = 8.66$$



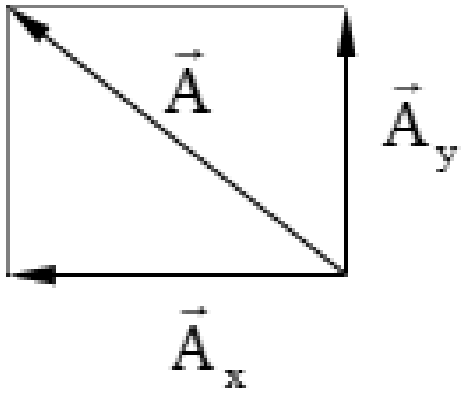
Vector Addition: Example 4

A boat moves northwest at a speed 10 km/h in a river flowing at 3 km/h eastward, what is the magnitude and direction of the boat's speed relative to the ground?

solution:

1- Represent the speed of the boat in the vector A , and the velocity of the river as the vector

2-Analyze the components of A :



$$A_y = A\sin 45 = 10 \times 0.707 = 7.07$$

$$A_x = A\cos 45 = 10 \times 0.707 = 7.07$$

For $B_y = 0 \rightarrow \; B = B_x = 3$

Vector Addition: Example 4 (continued)

3-We add the components in each direction separately, to get the two perpendicular components as follows:

$$\vec{R}_x = \vec{A}_x + \vec{B}_x$$

$$\vec{R}_x = -7.07 + 3 = -4.07$$

$$\vec{R}_y = \vec{A}_y + \vec{B}_y = 7.07 + 0 = 7.07$$

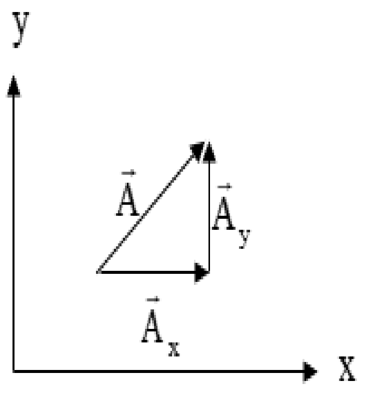
4- We apply the Pythagorean theorem to get the resultant R:

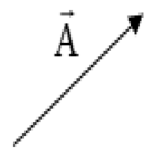
$$tan\theta = \frac{R_y}{R_x} \rightarrow \theta = tan^{-1}\frac{R_y}{R_x} \Longrightarrow \theta = 60^o$$

the angle from +x direction is : 180-60 =120°

Vector Addition: Unit Vectors

- If we have the vector A , it can be written as the vector addition of two components in the x and y direction as follows:





$\vec{A} = \vec{A}_x + \vec{A}_y$

- We obtained $\vec{A}_x$ and $\vec{A}_y$ as shown in the figure from a projection of $\vec{A}$ on the *x* and *y* directions respectively, and this is how to write a vector by analyzing it to its original components.

- The vector $\vec{A}_x$ can be written on the form: $\vec{A}_x = A_x\hat{\imath}$

Where $|\vec{A}_x| = A_x$ represents the magnitude (length) of the vector $\vec{A}_x$.$\hat{\imath}$ is called the unit vector along the direction (x), and it is parallel to the x direction, and its magnitude is one unit, that is: $|\hat{\imath}|=1$

Vector Addition: Unit Vectors

- So the unit vector does not affect the magnitude (or length) of the vector, it only determines its direction.
- In a similar manner ,we can write : $\vec{A}_y = A_y\hat{j}$

Where $|\vec{A}_y| = A_y$ represents the magnitude (length) of the vector $\vec{A}_y$.$\hat{j}$ is called the unit vector along the direction (y), and it is parallel to the y direction, and its magnitude is one unit, that is: $|\hat{j}| = 1$

A unit vector can be defined as a mathematical tool for determining the direction of a vector only, and it has nothing to do with the magnitude or length of the vector.

Substituting for $\vec{A}_x$ and $\vec{A}_y$ in equations we get:

$$\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{\imath} + A_y\hat{j}$$

Vector Addition: Unit Vectors: Example 5

Example 5:

Write the components of the following vector $\vec{A}$, then determine the vector in two ways: graphical and trigonometric methods, and check your solution by analyzing the vector into its components?

$\vec{A} = 2\hat{\imath} - 4\hat{\jmath}$

Solution :

we find that:

$\vec{A}_x = 2\hat{\imath}$, $\vec{A}_y = -4\hat{\jmath}$

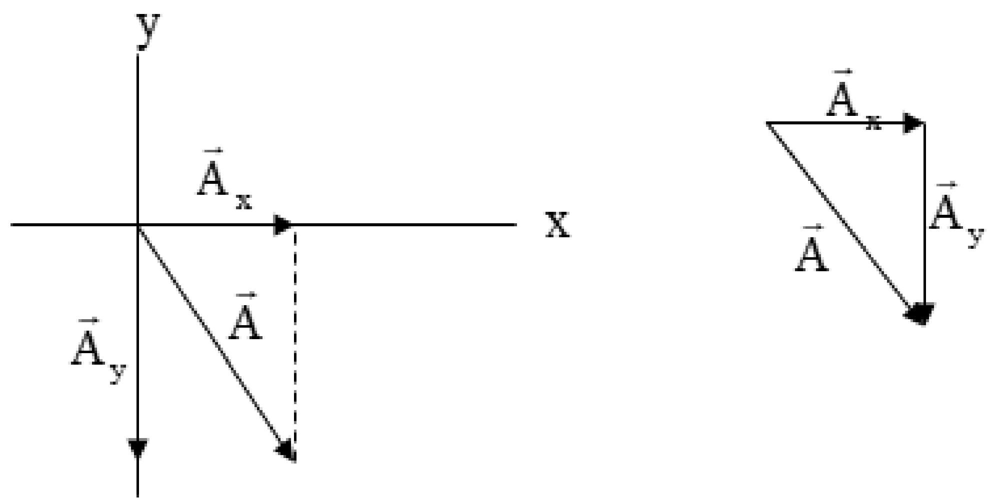
$A_x = 2$, $A_y = -4$

Vector Addition: Unit Vectors: Example 5

To determine the vector graphically, we proceed as follows:

1. We consider each unit = 1 cm
2. We calculate 2 cm in the positive direction of x, because $A_x = 2$:
3. We calculate 4 cm in the negative direction of y, because $A_y = -4$:
4. We determine the vector $\vec{A}$, which represents the resultant of the two vectors $\vec{A}_x$ and $\vec{A}_y$

by referring to the graphical method,
as shown in the figure:



Vector Addition: Unit Vectors: Example 5

To determine the vector by the trigonometric method, we do the following:

1- Determine the magnitude:

$$A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{2^2 + (-4)^2} = 4.48$$

2- Determine the direction:

$$tan\theta = \frac{A_y}{A_x} \rightarrow \theta = tan^{-1}\frac{A_y}{A_x} = tan^{-1}\frac{-4}{2} \Longrightarrow \theta = -63^o$$

$\Theta = -63^\circ$ (reference angle ;acute angle in the 4^{th} quadrant)

the angle from $+x$ is: $\varphi = 360 - 63 = 297^\circ$ (principal angle)

We can verify the solution by analyzing the vector into its components,

$A_x = A\cos\theta = 4.48 \cos -63 = 2$

$A_y = A\sin\theta = 4.48 \sin -63 = -4$

Relationship Between Reference Angle (θ) and Principal Angle (ϕ)

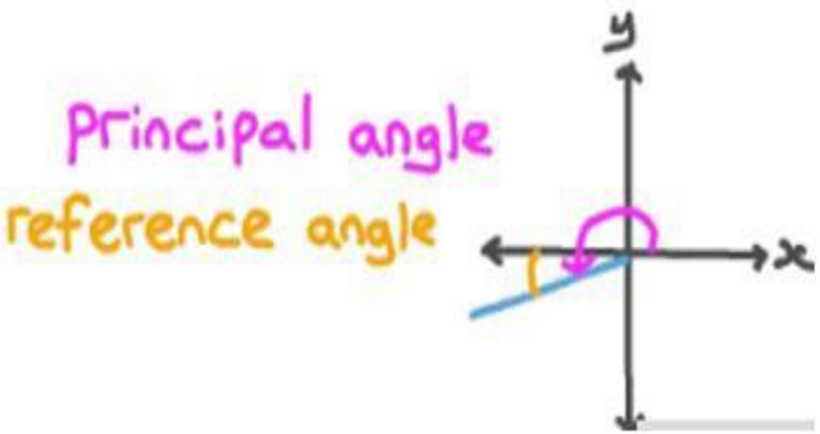
Relationship Between *Reference Angle* (θ) and *Principal Angle* (φ)

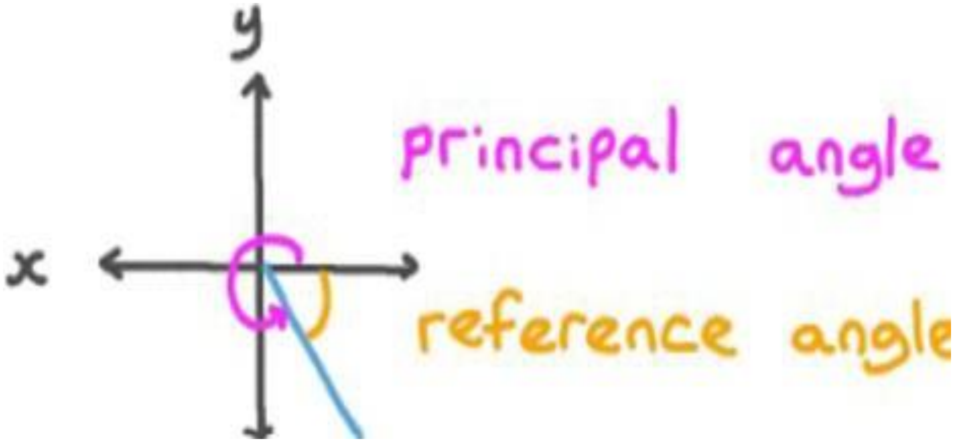
Let θ represent the reference angle (acute angle in the quadrant), and let φ represent the principal angle. The relationships between them in the four quadrants are given below:

Quadrant I $\varphi = \theta$; Quadrant II $\varphi = 180° - \theta$

Quadrant III $\varphi = 180° + \theta$; Quadrant IV $\varphi = 360° - \theta$

Same formulas apply when angles are given in radians. Replace 180° with π and 360° with 2π.





Vector Addition: Unit Vectors: Example 6

Example 6:

Write the components of the vector $\vec{B}$,then define the vector in graphical and trigonometric methods. Then, verify the solution by analyzing the vector into its components.

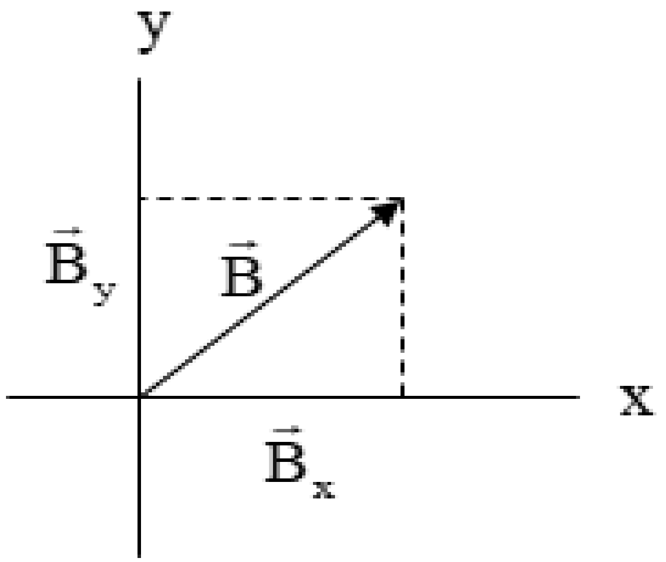
$\vec{B} = 2\hat{\imath} + 2\hat{\jmath}$

Solution:

we can find that:

$\vec{B}_x = 2\hat{\imath}$, $\vec{B}_y = 2\hat{\jmath}$

$B_x = 2$, $B_y = 2$



Following the same method as the previous example, we define the vector graphically as shown in the figure.

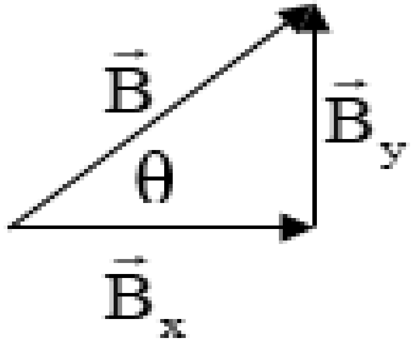
Vector Addition: Unit Vectors: Example 6

Follow next steps:

1- Determine the magnitude:

$$B = \sqrt{B_x^2 + B_y^2}$$

$$= \sqrt{2^2 + (2)^2} = 2.82$$



2- Determine the direction :

$$tan\theta = \frac{B_y}{B_x} \to \theta = tan^{-1}\frac{B_y}{B_x} = tan^{-1}\frac{2}{2} \Longrightarrow \theta = 45^o$$

We can verify the solution by analyzing the vector into its components,

$B_x = B\cos\theta = 2.82 \cos 45 = 2$

$B_y = B\sin\theta = 2.82 \sin 45 = 2$

Vector Addition: Unit Vectors: Example 7

Example 7:

Find the vector addition of the two vectors $\vec{A}$ and $\vec{B}$ mentioned in the previous two examples, by the graphical and components method .

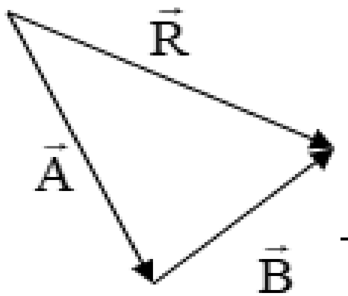
Solutions:

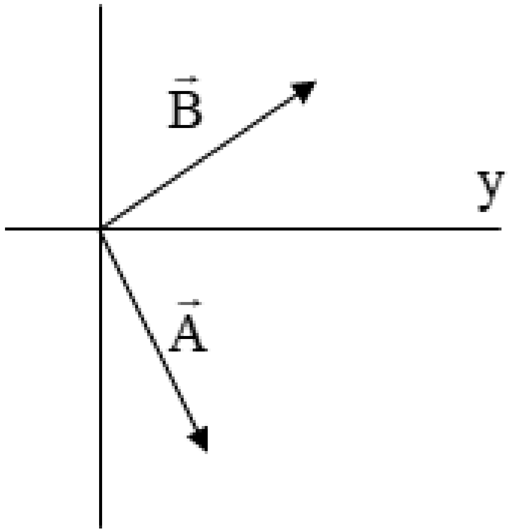
graphical method :

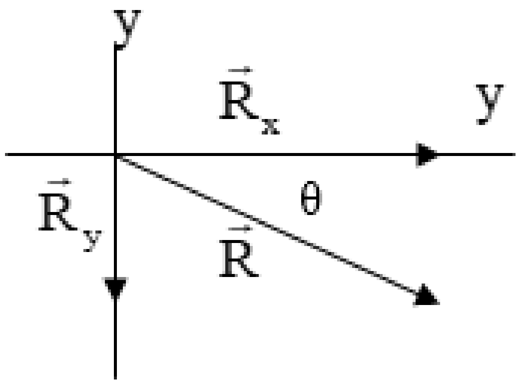
Suppose that the resultant of the two vectors $\vec{A}$ and $\vec{B}$ is the vector $\vec{R}$ so that:

$$\vec{R} = \vec{A} + \vec{B}$$

We define the vector $\vec{R}$ graphically as shown in the figure.







Vector Addition: Unit Vectors: Example 7

Components method :

analyzing $\vec{R}$ into its components:

$$\vec{R} = \vec{R}_x + \vec{R}_y$$
$$= R_x\hat{\imath} + R_y\hat{\jmath}$$
$$\Rightarrow R_x = (A_x + B_x) = 2 + 2 = 4$$
$$\Rightarrow R_y = (A_y + B_y) = -4 + 2 = -2$$
$$\Rightarrow \vec{R} = 4\hat{\imath} - 2\hat{\jmath}$$

1- To determine the vector magnitude :

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{4^2 + (-2)^2} = 4.47$$

2- the direction : $tan\theta = \dfrac{R_y}{R_x} \rightarrow \theta = tan^{-1}\dfrac{R_y}{R_x} = tan^{-1}\dfrac{-2}{2} \Longrightarrow \theta = -26.6^o \Longrightarrow \varphi = 333.7^o$

Vector Addition: Unit Vectors: Example 8

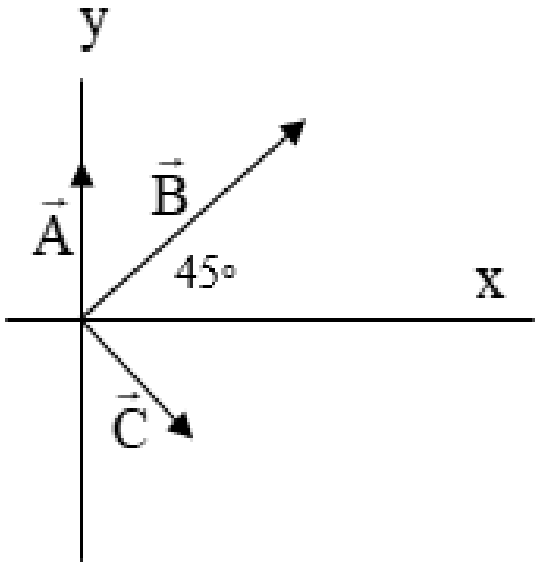
Given the following :

$$|\vec{A}| = 20$$
$$|\vec{B}| = 40$$
$$|\vec{C}| = 30$$

Calculate :

1 - The x and y components of the resultant of the three vectors.

2 - Magnitude and direction of the resultant.



$A_x = A\cos\theta = 20\cos 90 = 0$

$A_y = A\sin\theta = 20\sin 90 = 20$

$B_x = B\cos\theta = 40\cos 45 = 28.3$

$B_y = B\sin\theta = 40\sin 45 = 28.3$

$C_x = C\cos\theta = 30\cos 45 = 21.21$

$C_y = C\sin\theta = 30\sin 45 = -21.21$

Vector Addition: Unit Vectors: Example 8

Let the resultant vector $\vec{R}$, where:

$$\vec{R} = \vec{\mathrm{R}}_x + \vec{\mathrm{R}}_y$$
$$= \mathrm{R}_x\hat{\imath} + \mathrm{R}_y\hat{\jmath}$$
$$\Rightarrow \mathrm{R}_x = (\mathrm{A}_x + \mathrm{B}_x + \mathrm{C}_x) = 0 + 28.3 + 21.21 = 49.5$$
$$\Rightarrow \mathrm{R}_y = \left(\mathrm{A}_y + \mathrm{B}_y + \mathrm{C}_y\right) = 20 + 28.3 - 21.21 = 27.1$$
$$\Rightarrow \vec{R} = 19.5\hat{\imath} + 7.12\hat{\jmath}$$

The magnitude of $\vec{R}$ is :

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(49.5)^2 + (27.1)^2} = 56.4$$

The direction of $\vec{R}$ is :

$$\tan\theta = \frac{R_y}{R_x} \rightarrow \theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{27.1}{49.5} \Longrightarrow \theta = 28.7^o$$

Thanks

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Phys 105

Physics for Architecture Students

Mechanics

Chapter 2

Motion in one Dimension

Uniform Motion Equations in One Dimension

One dimensional motion is motion along a straight line. The line used for motion is usually either x direction (a car moving on a straight road) or y direction (free falling stone).

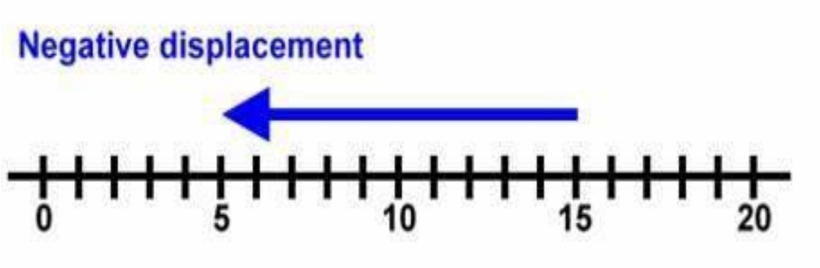
In the following, basic description to the motion of an object in one dimension will be given. In particular, formulas will be shown that relate the basic quantities of motion: *displacement*, *velocity* and *acceleration*.

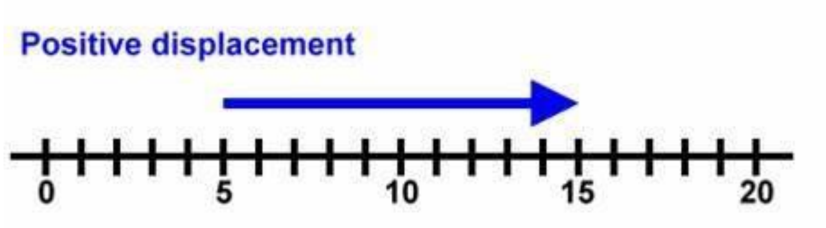
Displacement (Δx) :

The displacement of an object is defined as the change in its position.

$$\overrightarrow{\Delta x} = x_f - x_i$$

Where: x_i is initial position and x_f is final position





Uniform Motion Equations in 1-D: Average Velocity

Displacement can be +ve , -ve or zero.

Average Velocity (v):

When an object travels a displacement $\overrightarrow{\Delta x}$ within a time of t, and its initial velocity v_i and final velocity v_f , then the average velocity is given by the relationship:

Average velocity = displacement/time

$$\bar{v} = \frac{\overrightarrow{\Delta x}}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

Where x_i and x_f denote the initial final position respectively ; t_i and t_f denote the corresponding time for the initial final position .

Velocity is a vector quantity and its unit is : m/s

Uniform Motion Equations in 1-D: Instantaneous Velocity

Instantaneous velocity (v) :

It is the velocity of a body at a given moment, and is given by the relationship:

$$v = \frac{dx}{dt}$$

Average Speed:

Average speed is defined as:

$$v_{avg} = \frac{total\ distance}{total\ time}$$

Instantaneous Speed:

Instantaneous speed is defined as:

$$|v| = \left|\frac{dx}{dt}\right|$$

Uniform Motion Equations in 1-D: Example 1

Example 1: If the meter reading of a car was 22,687 km at the beginning of a trip, then the reading was 22,791 km at the end of the 4 h trip. Calculate the average speed of the car in SI units given that the car was traveling in a straight line.

The solution:

$$\because v_{avg} = \frac{total\ distance}{total\ time}$$
$$= \frac{22791 - 22687}{4}$$
$$= 26\,\frac{km}{h}$$
$$= \frac{26 \times 1000}{3600}$$
$$= 7.2\ m/s$$

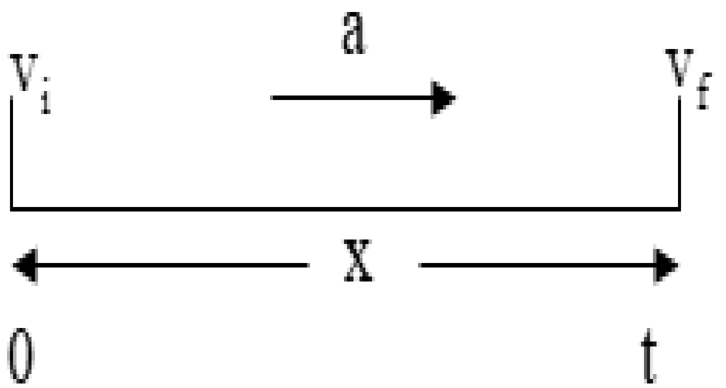
Uniform Motion Equations in 1-D: Acceleration

Acceleration (a):
It is the rate of change of velocity with respect to time. Acceleration is positive when the velocity increases, and negative when the velocity decreases. It is a vector quantity and its unit is m/s^2. We will limit our study to a **uniform acceleration**, where the rate of change of velocity is constant with respect to time.
Acceleration = change in velocity /time

Average Acceleration:

$$\bar{a} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

Instantaneous Acceleration: $a = \dfrac{dv}{dt}$



where v_i and v_f represent the initial and the final velocity of the body respectively.

Uniform Motion Equations in 1-D: Acceleration

- An object is moving with uniform acceleration when its velocity changes at a constant rate with respect to time. A change in velocity occurs either by changing its magnitude, changing its direction, or both.
- To write equations of uniform motion in one dimension, suppose that a car is moving with an initial velocity and cuts a displacement *x* within a time *t* at which its final velocity becomes v. If the acceleration of the car is *a*, the following equations of motion can be written by which we can determine the unknown quantity in the question if the last quantities are known.

$$v_f = v_i + at \qquad (1)$$

$$\bar{v} = \frac{v_i + v_f}{2} \qquad (2)$$

$$x = v_i t + \frac{1}{2} a t^2 \qquad (3)$$

$$v_f^2 = v_i^2 + 2ax \qquad (4)$$

Uniform Motion Equations in 1-D: Example 2

- **Example 2:** A car moving with an initial speed (20 m/s) , and acceleration -1 m/s^2. Calculate its velocity after 10 s.

Solution:

$v_i = 20\ \mathrm{m/s}$ $\quad a = -1\ \mathrm{m}/s^2$ and t= 10 s

$\because\ v_f = v_i + at$

$\rightarrow\ v_f = 20 - 1 \times 10$

$\rightarrow\ v_f = 10$ m/s

Uniform Motion Equations in 1-D: Example 3

- **Example 3:** A car moving from rest has an acceleration of 5 m/s^2. Calculate (a) the distance traveled and (b) its speed after 4 s

Solution:

$v_i = 0 \text{ m/s}$ $\quad a = 5 \text{ m}/s^2$ and t= 4 s

(a) $x = v_i t + \frac{1}{2} a t^2$

$$x = 0 + \frac{1}{2}(5)(4)^2$$

$\rightarrow$ x = 40 m

(b) $v_f^2 = v_i^2 + 2ax$

$$v_f^2 = 0 + 2(5)(40) = 400 \rightarrow v_f = 20\, m/s$$

Uniform Motion Equations in 1-D: Example 4

Example 4:

A car starts from rest and accelerates uniformly until its velocity is 5 m/s in 10 s. Find (a) the acceleration of the car and (b) the distance covered during this time?

Solution:

$v_i = 0\ \mathrm{m/s}$ $\quad$ $v_f = 5\ \mathrm{m/s}$ and t= 10 s

(a) $v_f = v_i + at$

$$\rightarrow a = \frac{v_f - v_i}{t} = \frac{5-0}{10} = 0.5\ m/s^2$$

(b) $x = \frac{v_f + v_i}{2} t = \frac{5+0}{2} \times 10 = 25\ m$

Free Fall

The equations of uniform motion in one dimension can be applied to bodies in free fall after replacing the value of acceleration a in the equations with gravitational acceleration g whose value is equal 9.8 m/s^2 and its direction is always downwards toward the center of the earth. So the equations are:

$$v_f = v_i - gt \qquad (1)$$

$$y = v_i t - \frac{1}{2} g t^2 \qquad (2)$$

$$v_f^2 = v_i^2 - 2gy \qquad (3)$$

$v \uparrow +\ \ and\ v \downarrow -$

$a\ \ is\ always = -9.8\, m/s^2$

y above firing level is + and below firing level is -

Free Fall: Example 5

Example 5:

A stone falls from a 450 m high building. Neglecting air resistance, calculate:

(a)The time required for the stone to reach the ground.

(b)The speed of the stone when it hits the ground.

Solution

v_i=0 m/s, $y = -450\ m$

(a) $\because\ y = v_i t - \frac{1}{2} g t^2$

$$\Longrightarrow y = 0 - \frac{1}{2} g t^2$$

$$t^2 = -\frac{2y}{g} = -\frac{2(-450)}{9.8} \Longrightarrow t = \sqrt{\frac{2(450)}{9.8}} = 9.6\ s$$

(b) $v_f = v_i - gt = 0 - 9.8(9.6) = -94 \downarrow$ (downward)

Free Fall: Example 6

Example 6:

A stone is thrown from the ground to the top with an initial velocity of 16 m/s.

Calculate:

(a) The maximum height that will be reached by the stone.

(b) The time required for the stone to return to the ground.

Solution v_i=+16 m/s ↑, v_f=0 m/s

$$(a) \because v_f^2 = v_i^2 - 2gy$$

$$\longrightarrow 0 = (16)^2 - 2gy$$

$$y = \frac{(16)^2}{2g} = 13\ m$$

(b) $v_f = v_i - gt$

$$\longrightarrow t = -\frac{v_f - v_i}{g} = -\frac{-16 - 16}{9.8} = 3.27\ s$$

Notes:

time up ↑ = time down ↓ (to same level)

Velocity down ↓ = - Velocity up ↑

Velocity has same magnitude at same level

Thanks

King Saud University

College of Science

Phys 105

Physics for Architecture Students

Mechanics

Chapter 4

Newton's laws of motion

Newton's laws of motion: Inertia and Mass

Inertia:

It is the property possessed by the body to resist a change in its state of rest or uniform motion in a straight line. That is, the body is inert or unable by itself to change its position at rest or moving with a uniform motion along a straight line, and an external force must be applied on it to change its original state.

Mass (m):

It is the quantitative measure of a body's inertia. If the body's resistance to changing its static or kinetic state is great, it means that its mass is large. Mass is measured in kg.

Newton's laws of motion: Force

Force:

It is an effect that may lead to a change in the kinetic state of the body and is indicated by the presence of acceleration that changes the value or direction of the body's velocity, and the force is measured by the SI unit which is Newton : N.

An example of force is the gravitational force acting on the body, which is equal to the weight of the body w, where:

$$w = mg$$

$g = 9.8\ m/s^2$ is the gravitational acceleration.

Newton's laws of motion: Newton's First Law

"A body remains at rest or moving at a constant speed unless acted upon by a net external force not equal to zero".

In mathematical form, we write:

$$\sum F = 0$$

Where $\sum F$ represents the net forces acting on the body.

Object is said to be at equilibrium if any of the following is True:

1. $\sum F = 0$
2. $v = constant$
3. $a = 0$

v and a are its velocity and acceleration respectively.

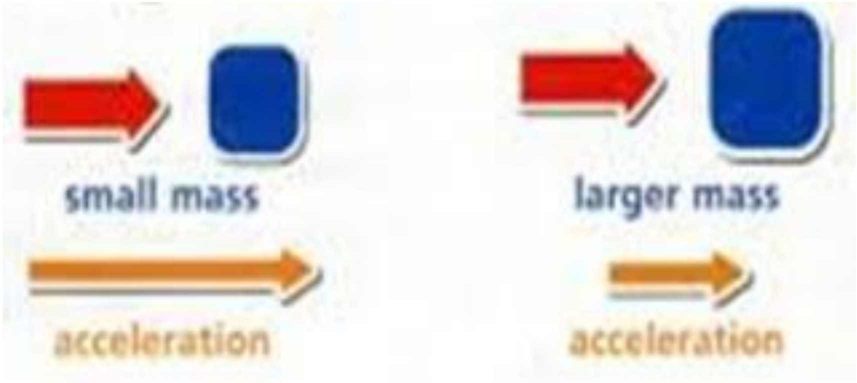
Newton's laws of motion: Newton's second law

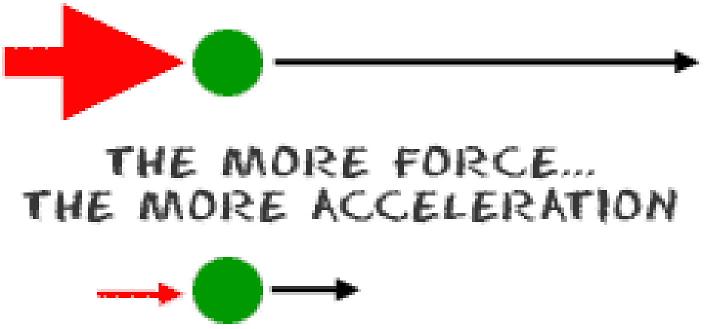
The net force acting on the body is directly proportional to the mass of the body and its acceleration, and the direction of the resultant force is the same as the direction of the body's acceleration

In mathematical form, we write:

$$\sum F = ma$$

$$\Rightarrow \sum F_x = ma_x \qquad \text{and} \qquad \sum F_y = ma_y$$





Newton's laws of motion: Example 1

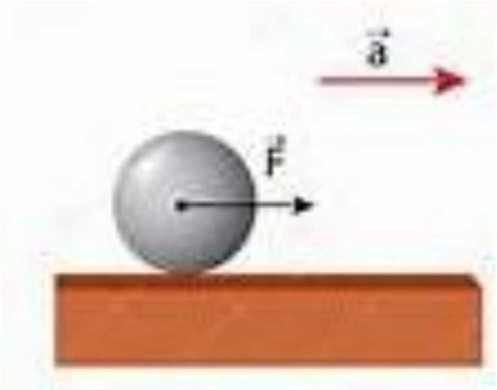
Example 1:

What force is needed to get an acceleration of 6 m/s^2 for a 5 kg mass?

Solution

$$\sum F = ma$$

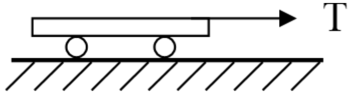
$$F = 5 \times 6$$
$$\therefore F = 30\ N$$



Newton's laws of motion: Example 2

Example 2:

In the shown figure, what tension is required in the rope to pull the cart with an acceleration 0.5 m/s^2 if the *weight* of the cart is 80 N.



Solution

Draw a diagram of the forces acting on the cart, then apply Newton's 2nd law:

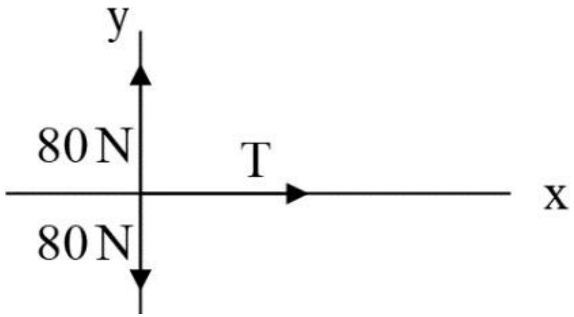
y: $\sum F_y = 0$

x: $\sum F_x = ma_x$

$\Rightarrow \quad F_x = T = ma_x \qquad (1)$

$\because w = mg \;\Rightarrow m = \frac{w}{g} = \frac{80}{9.8} = 8.16\,kg \qquad (2)$

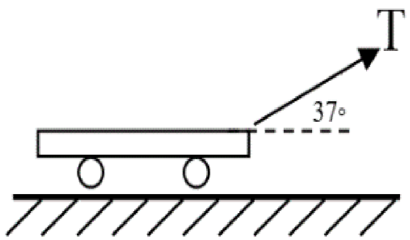
(2) in (1): $T = ma_x$ =8.16×0.5=4.1 N



Newton's laws of motion: Example 3

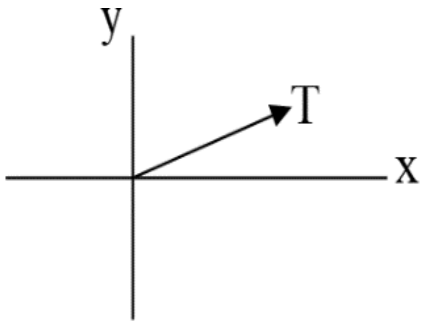
Example 3:

A cart of weight 4 kg is pulled with an inclined force of 6 N and makes an angle of 37° with the horizontal plane as shown in the following figure.
What is the acceleration of the cart?



Solution

There is no movement along y-direction → We apply Newton's 2^{nd} law:



$$y: \quad \sum F_y = 0$$
$$x: \quad \sum F_x = ma_x$$
$$\Rightarrow \quad F_x = T\cos 37 = ma_x$$
$$\therefore \quad 6\cos 37 = 4\,a_x$$
$$\Rightarrow \quad a_x = \frac{6\cos 37}{4} = 1.2\ m/s^2$$

Newton's laws of motion: Example 4

Example 4:

In the figure, mass A = 12 kg and mass B = 30 kg are placed on a frictionless table. Calculate: (a) the group acceleration and (b) the tension in the rope.

Solution

There is no movement along y-direction → We apply Newton's 2nd law:

Steps:

1- Select motion direction

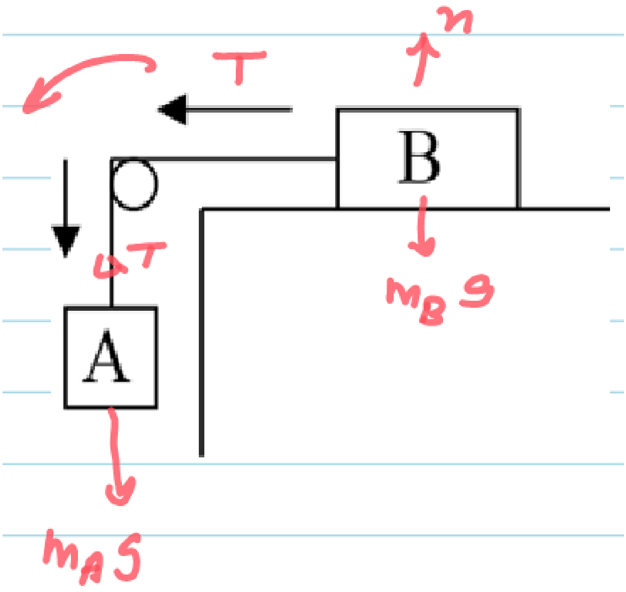
2- Write down the free form forces

3- Apply Newtons' 2nd law for each mass

4- Solve Equations

5- Find either a or T

6- Substitute to find the other quantity



Newton's laws of motion: Example 4

Solution

1- Select motion direction: Counter Clockwise

2- Write down the free form forces: Shown on the figure

3- Apply Newtons' 2nd law for each mass:

$m_A: m_A g - T = m_A a$ (1)

$m_B: \quad T = m_B a$ (2)

4- Solve Equations:

(1) + (2) $\Rightarrow m_A g = (m_A + m_B)a$ (3)

5- Find either a or T:

(3) $\Rightarrow a = \frac{m_A g}{m_A + m_B} = \frac{12\times 9.8}{12+30} = 2.8\, m/s^2$ (4)

6- Substitute to find the other quantity:

(4) In (2): $\Rightarrow T = m_B a = 30 \times 2.8 = 84\ N$ (5)

Newton's laws of motion: Example 5

Example 5:

Calculate the acceleration for the group shown in the figure, where mass A = 12 kg and mass B = 30 kg . Then calculate the tension in the thread?

Solution

We implement same steps as in Example 4:

Steps:

1- Select motion direction

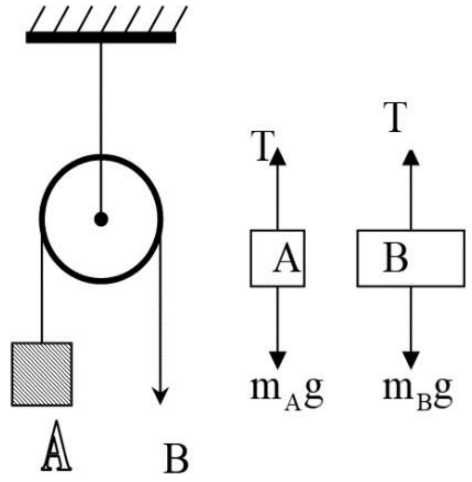
2- Write down the free form forces

3- Apply Newtons' 2^{nd} law for each mass

4- Solve Equations

5- Find either a or T

6- Substitute to find the other quantity



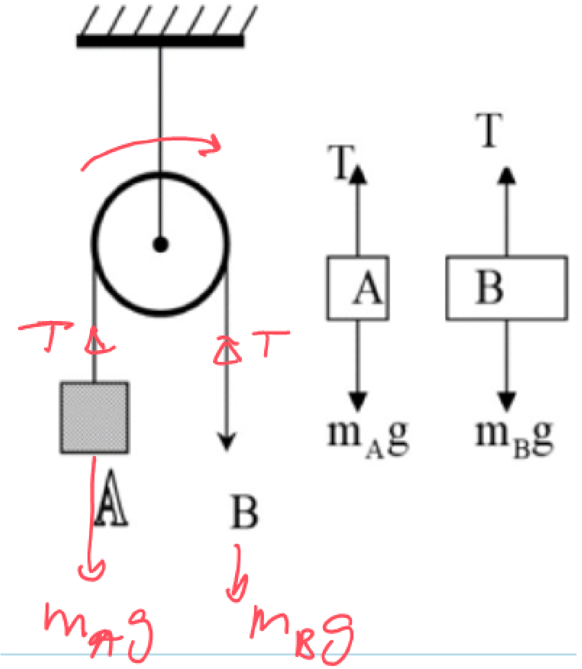
Newton's laws of motion: Example 5

Solution

1- Select motion direction: Clockwise

2- Write down the free form forces: Shown on the figure

3- Apply Newtons' 2nd law for each mass:



$$m_B : m_B g - T = m_B a \qquad (1)$$

$$m_A :\ T - m_A g = m_A a \qquad (2)$$

4- Solve Equations:

$$(1) + (2) \Rightarrow m_B g - m_A g = (m_A + m_B)a \qquad (3)$$

5- Find either a or T:

$$(3) \Rightarrow a = \frac{m_B g - m_A g}{m_A + m_B} = \frac{30\times 9.8 - 12\times 9.8}{12+30} = 4.2\ m/s^2 \qquad (4)$$

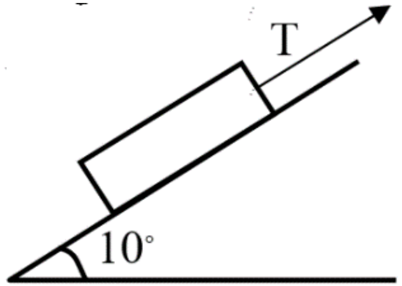
6- Substitute to find the other quantity:

$$(4) \text{ In } (2): \Rightarrow T = m_A a + m_A g = 12 \times 4.2 + 12 \times 9.8 = 168\ N$$

Newton's laws of motion: Example 6

Example 6:

A box of mass 1000 kg is being pulled up a surface inclined at an angle = θ by a rope with a tension of 2000 N. Calculate the maximum acceleration of the box?



Solution

We analyze the forces acting on the box: There is no movement in the perpendicular direction on the inclined plane →

y: $\sum F_y = 0$

x: $\sum F_x = ma_x$

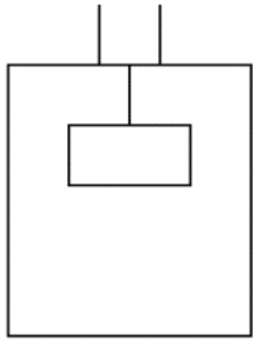
$\Rightarrow \quad T - mgsin\theta = ma_x$

$\Rightarrow \quad a_x = \frac{T - mgsin\theta}{m} = \frac{2000 - 1000 \times 9.8 \times sin10}{1000} \approx 0.3\ m/s^2$

Newton's laws of motion: Example 7

Example 7:

A body of mass 30 kg is suspended from a rope hanging from the ceiling of an elevator, as shown in the figure. What is the tension in the rope if the elevator is accelerating at the rate of: (a) $4 m/s^2$ Upwards . (b) $4 m/s^2$ downwards.



Solution

We analyze the forces acting on the body, and write the relationship: →

y: $\quad \sum F_y = ma_y$

(a) $\Rightarrow T - mg = ma_y \qquad (1)$

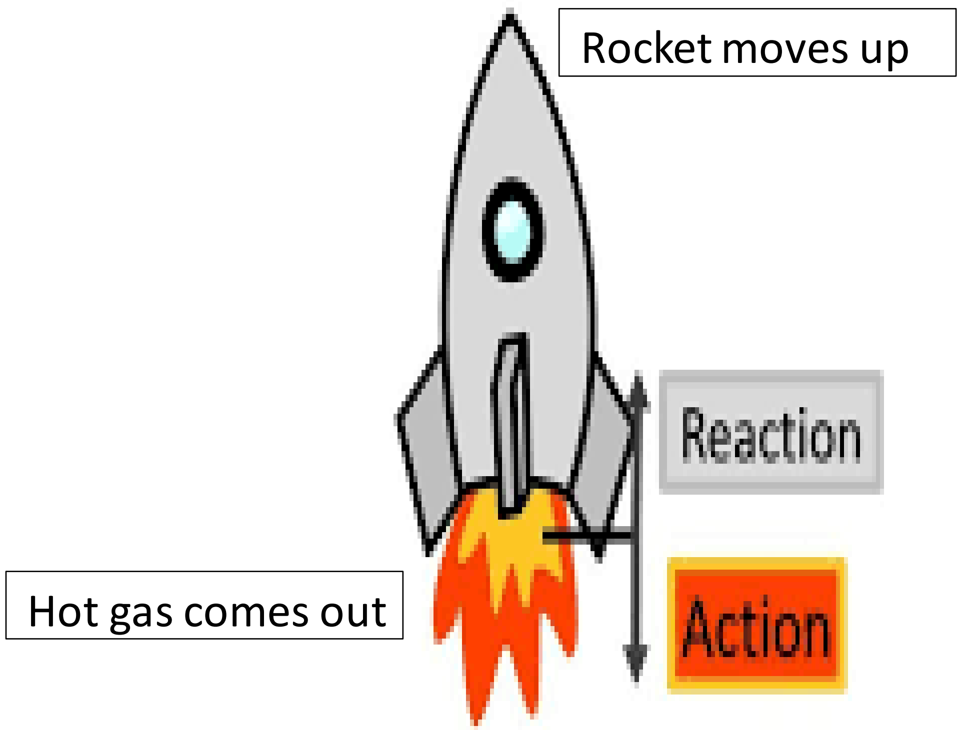
$\Rightarrow \quad T = mg + ma_y = 30 \times 9.8 + 30 \times 4 = 418\ N$

(b) (1) becomes: $T - mg = -ma_y$

$\Rightarrow \quad T = mg - ma_y = 30 \times 9.8 - 30 \times 4 = 174\ N$

Newton's laws of motion: Newton's third law

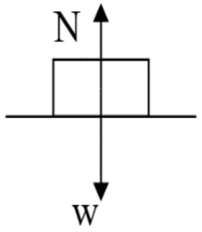
If a body exerts a force on a second body, then the second body acts on the first with a force equal in magnitude and opposite in direction. One of the forces is called the action force and the other is called the reaction force.

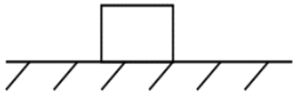


Newton's laws of motion: Example 8

Example 8:

The block shown in the adjacent figure is placed on a table top. Determine the action force and the reaction force if the weight of the mass is w.





Solution

We analyze the forces acting on the two bodies, the mass and the table: →

The mass acts on the table with a force that is the weight of the mass w and its direction downward, and this is the force of the action.

The table acts on the mass with a force N equal to the mass of the mass and its upward direction. N is called the normal force or it represents the reaction force.

Newton's laws of motion: Example 9

Example 9:

A rope attached to the left is pulled from the right by the hand. Determine the action and reaction forces between the hand and the rope.

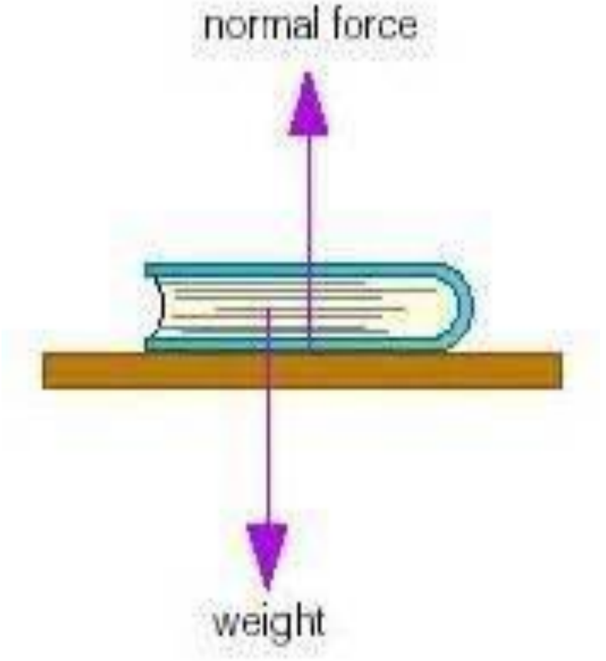
Solution

If we consider the force of the hand pulling the rope towards the right and its value T (action force), then the rope exerts a force of T on the hand also but towards the left (reaction force).

Newton's laws of motion: Normal Force (N)

Normal force is a contact force applied by a surface on an object. Normal force is the force that the ground (or any surface) pushes back up when an object is pushing on it. The normal force always has a 90-degree angle with the surface that applies it. For example, if a book is resting upon a surface, then the surface is exerting an upward force (normal force) upon the book in order to support the weight of the book.

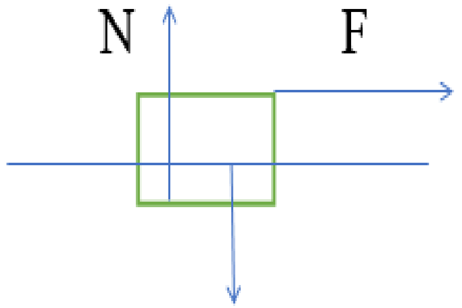
The normal force isn't necessarily equal to the force due to gravity (w weight) ; it's the force perpendicular to the surface when an object is pressing on it. The normal force can be calculated using Newton's Second Law.



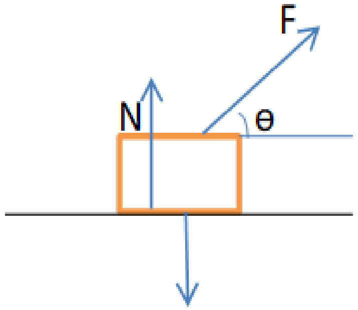
Newton's laws of motion: Examples 10

Examples 10:

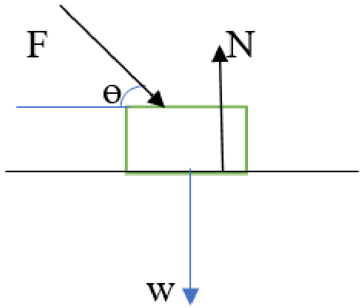
The magnitude of the Normal force **N** changes according to the studied cases. Here is a list of use cases examples:



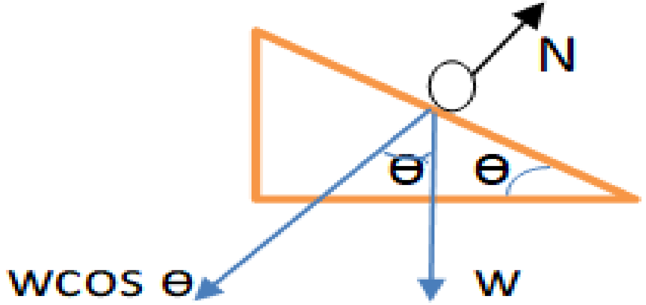
$N = w$



$N = w - F\sin\theta$



$N = w + F\sin\theta$



$N = w\cos\theta$

Newton's laws of motion: Normal Force (N)

- **Normal Force (N):** A contact force exerted by a surface on the object placed upon it.
- **Direction**: It is always at a 90-degree angle to the surface (perpendicular to the surface).
- **Example**: When a book rests on a table, the table exerts an upward normal force that supports the weight of the book.
- **Not always equal to the weight of the object**: The normal force is not necessarily equal to the gravitational force; rather, it is the perpendicular force from the surface caused by the object pressing on it.
- **Calculation**: It can be calculated using Newton's Second Law in the direction perpendicular to the surface.

Newton's laws of motion: Frictional force

It is the force that appears between two adjacent surfaces to impede the movement between them, and its direction is always opposite to the direction of the movement. We will focus on the frictional friction that occurs during movement only.

The force of friction increases with the increase in the perpendicular force that one surface exerts on the other, that is:

$$F_f \alpha N$$

$$F_f = \mu N$$

Where μ is the coefficient of friction between the two contacting surfaces. The coefficient of friction depends upon the nature of the surfaces. The following table shows the values for the coefficient of kinetic friction for some contacting surfaces:

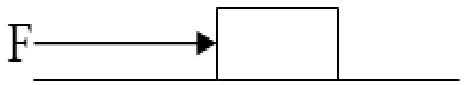
Newton's laws of motion: Frictional force

System	μ
Rubber on dry concrete	0.7
Rubber on wet concrete	0.5
Wood on wood	0.3
Waxed wood on wet snow	0.1
Metal on wood	0.3
Steel on steel (dry)	0.3
Steel on steel (oiled)	0.03
Teflon on steel	0.04
Bone lubricated by synovial fluid	0.015
Shoes on wood	0.7
Shoes on ice	0.05
Ice on ice	0.03
Steel on ice	0.02

Newton's laws of motion: Example 11

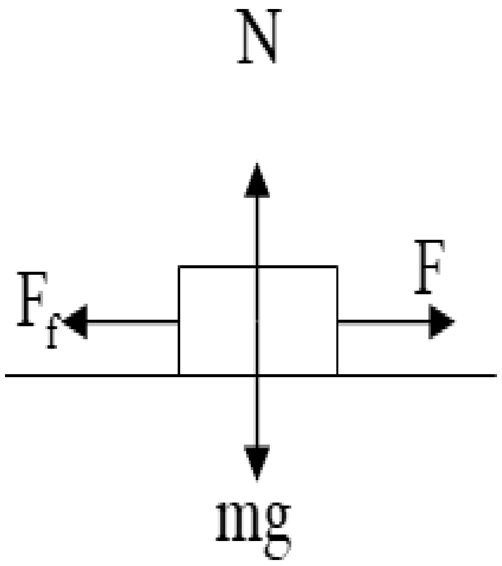
Example 11:

A wooden box of mass 100 kg is pushed against a surface with a force of 350 N. Calculate the acceleration of the box if the coefficient of friction is 0.3.



Solution

We analyze the forces acting on the box:



y: $\sum F_y = ma_y$

$\Rightarrow N - mg = 0 \Rightarrow N = mg = 100 \times 9.8 = 980\ N$

x: $\sum F_x = ma_x$

$\Rightarrow \ F - F_f = ma_x$

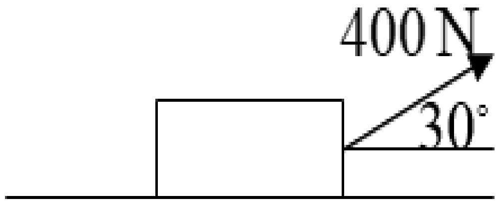
$$\Rightarrow \quad a_x = \frac{F - F_f}{m} = \frac{350 - \mu N}{100}$$

$$= \frac{350 - 0.3 \times 980}{100} = 0.56\ m/s^2$$

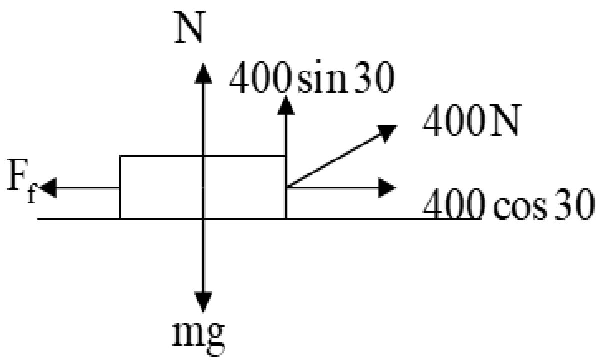
Newton's laws of motion: Example 12

Example 12:

A 70-kg box is pulled by a force of 400 N, as shown in the figure. If the coefficient of friction is 0.5, *calculate the acceleration of the box.*



Solution:



y: $\sum F_y = ma_y = 0$ (1)

$\Rightarrow N - mg = 0 \Rightarrow N = mg = 100 \times 9.8 = 980\ N$

x: $\sum F_x = ma_x$

$\Rightarrow\ F cos30 - F_f = ma_x$

$\Rightarrow\ F cos30 - \mu N = ma_x$ (2)

(1) $\Rightarrow: N + 400 sin30 - mg = 0$

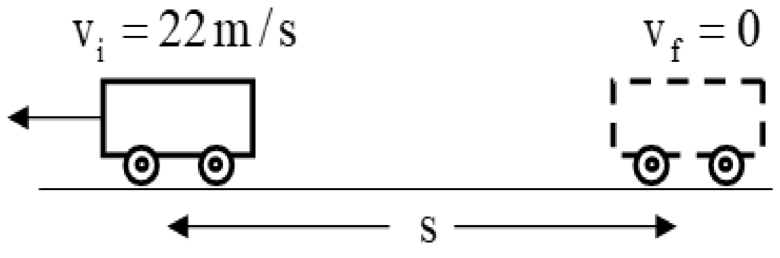
$\Rightarrow N = mg - 400 sin30 = 70 \times 9.8 - 400 \times 0.5 = 486\ N$ (3)

(3) In (2) $\Rightarrow a_x = \frac{F cos30 - \mu N}{m} = \frac{400 cos30 - 0.5 \times 486}{70} = 1.47\ m/s^2$

Newton's laws of motion: Example 13

Example 13:

A car is traveling at 22 m/s on a road with a coefficient of friction of 0.7. *How far does the car travel before it comes to a stop* when the car's brakes are fully applied to force it to stop?



Solution:

$$F_f = -\mu N = -\mu mg = ma_x$$
$$\Rightarrow a_x = -\mu g = -0.7 \times 9.8 = -6.9\,m/s^2$$
$$\because v_f = 0$$
$$\because v_f^2 = v_i^2 + 2a_x s$$

$$\Rightarrow s = \frac{v_f^2 - v_i^2}{2a_x} = \frac{0 - 22^2}{2(-6.9)} = 35\text{ m}$$

Thanks

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Phys 105

Physics for Architecture Students

Mechanics

Chapter 5

Work and Energy

Work and Energy: Work

Work:

The work done by a constant force is defined as the product of the force component in the direction of the displacement and the magnitude of that displacement. Work is a scalar quantity and its unit is joules (J):

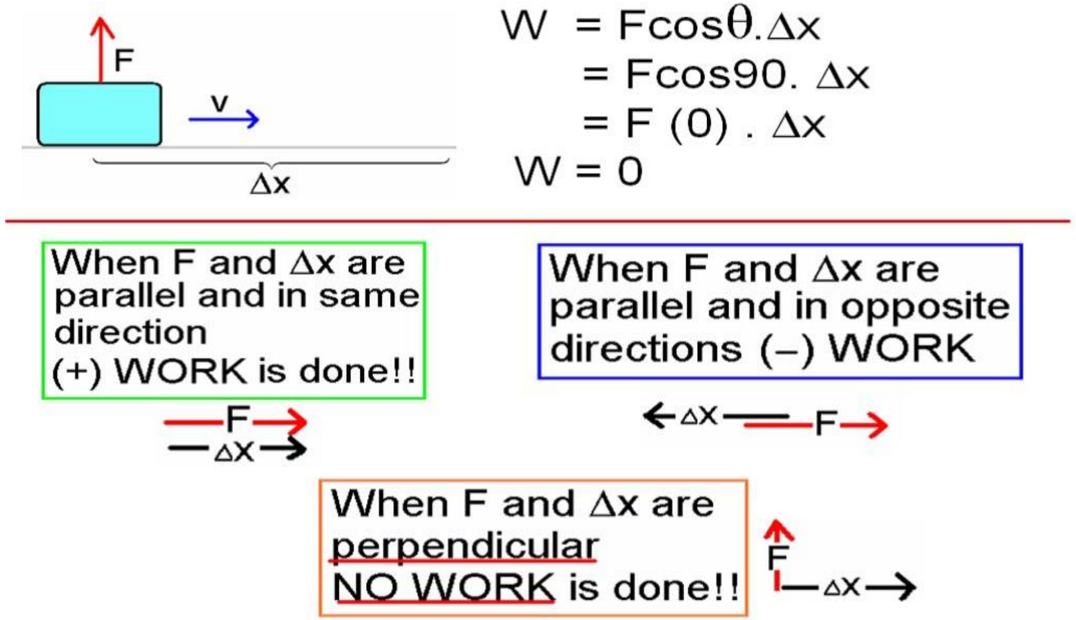
$J \equiv N.m.$

When a force of magnitude F acts on the body shown in the figure and displaces it by a distance Δx, the component of the force in the direction of the displacement is:

$$W = \mathbf{F}.\mathbf{\Delta x} = (F\cos\theta)\Delta x$$

And if F is *parallel* to s, $\theta=0$, then $\cos\theta = 1$

$\Rightarrow$ W = max, when $\theta=90$, $\cos\theta=0 \Rightarrow W = 0$

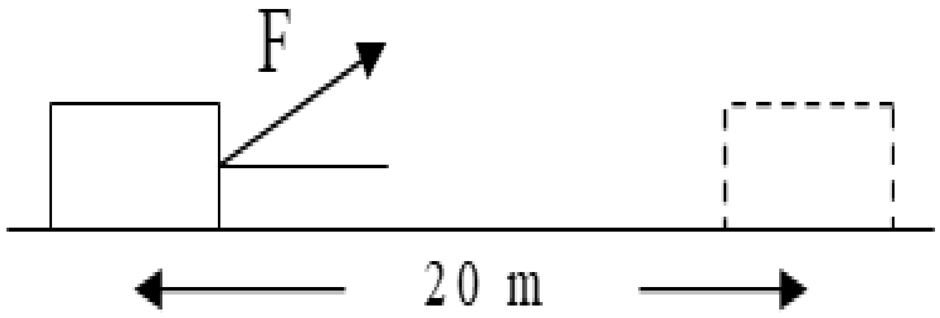


Work and Energy : Example 1

Example 1:

The object shown in the figure is pulled by a force F of 150 N, causing it to move with a displacement of 20 m. Calculate the work done on the body during this displacement.

Solution



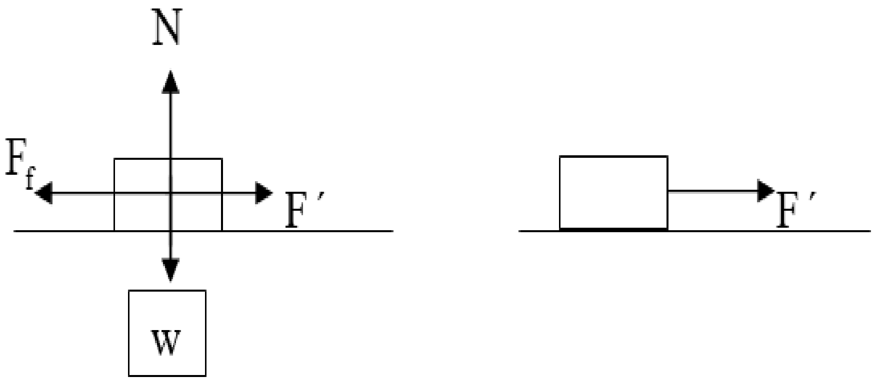
$$W = Fs\cos\theta$$
$$= 150 \times 20 \times \cos 30$$
$$= 2.6\ kJ$$

Work and Energy : Example 2

Example 2:

A 500 N box is pushed against a horizontal floor with a force of 250 N. If the coefficient of kinetic friction is 0.4, calculate the work done on the box to push it 20 m.

Solution



$$
\begin{aligned}
W &= (net\ force) \times s \times cos\theta \\
&= (F - F_f) \times s \times cos\theta \\
&= (250 - \mu.N) \times s \times cos\theta \\
&= (250 - \mu.m.g) \times s \times cos\theta \\
&= (250 - 0.4 \times 500) \times 20 \times cos0 \\
&= (250 - 200) \times 20 \times cos0 \\
&= 50 \times 20 \times 1 = 1000\,J
\end{aligned}
$$

Work and Energy: Power

Power:

It is the rate of work done by a force, or it is the work done by a force during a unit time, and power is a scalar quantity.

Average power = work/time

$$\bar{P} = \frac{W}{t}$$

The power is measured in watts (W): 1W = 1 J/s, and if the force that does the work = F.cosθ , then the work is equal to : W = F.s.cosθ , and therefore the average power is:

$$\bar{P} = \frac{Fscos\theta}{t} = F\bar{v}cos\theta$$

Where $\bar{v}$ represents the average velocity, and θ represents the angle between the direction of the velocity (movement) and the direction of the acting force.

Work and Energy : Example 3

Example 3:

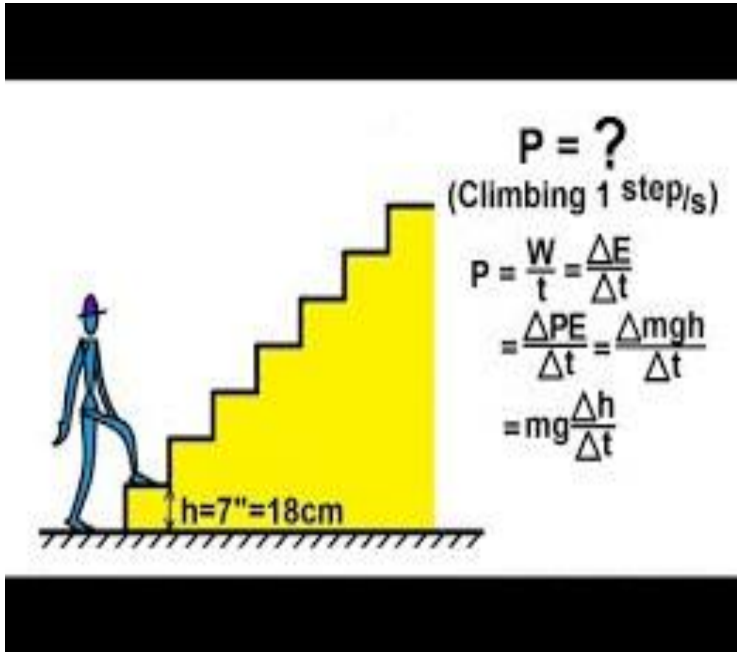
A 70-kg man climbs a ladder of 3m height in 2 s.

(a) How much work is done by the man do against the force of gravity?

(b) What is the average power of the man?

Solution

(a) Since the force of gravity on a man is his weight w, and it is equal to: w = mg, so the man needs to exert a constant force of F against gravity during his ascent to the top, meaning that: F = w = mg and this force that he needs to exert is in the same direction as the displacement to higher, that is, and therefore the required work is:



$$W = (70 \times 9.8) \times 3 \times cos0 = 2058\,J$$

Work and Energy : Example 3

Example 3:

A 70-kg man climbs a ladder of 3m height in 2 s.

(a) How much work is done by the man do against the force of gravity?

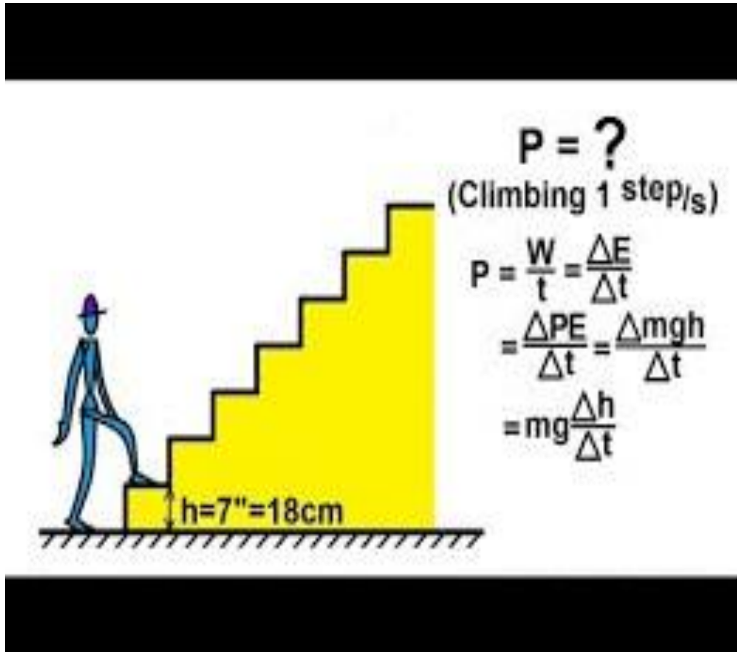
(b) What is the average power of the man?

Solution

(b)

$$\bar{P} = \frac{2058}{2}$$

$$= 1029\, W$$



Work and Energy : Example 4

Example 4:

A block of mass 250 kg is lifted by a winch at a constant speed of 0.1 m/s. What is the power exerted by the lever?

Solution

$$
\begin{aligned}
\bar{P} &= F\bar{v}cos\theta \\
&= (mg)\bar{v}cos\theta \\
&= (250 \times 9.8)(0.1)(1) \\
&= 245\ W
\end{aligned}
$$

Work and Energy : Example 5

Example 5:

A box of mass 200 kg is pulled on a horizontal surface with a coefficient of friction of 0.4 by an engine.

(a) What is the motor power required for the box to move at a constant speed of 5 m/s?

(b) What is the work done by the motor in 3 min?

Solution

(a) $\bar{P} = F\bar{v}$

The force required to move the box at a constant speed must be equal to the force of friction, that is:

$$\bar{P} = F_f\bar{v}$$
$$= \mu \times N \times v = \mu \times mg \times v$$
$$= 0.4 \times 200 \times 9.8 \times 5 = 3920\ W$$

(b) $\because \bar{P} = \frac{W}{t}$

$$\Longrightarrow W = \bar{P}t = 3920 \times 3 \times 60 = 705600 J = 705\ kJ$$

Work and Energy: Kinetic Energy

Kinetic Energy :

The kinetic energy of an object expresses the energy produced by its motion. If we have a body of mass *m* moving at a certain moment with a speed *v* in a straight line, then its kinetic energy *K* at that moment is:

$$K = \frac{1}{2} m v^2$$

Kinetic Energy

$$K = \frac{1}{2} m V^{2}$$

Kinetic Energy

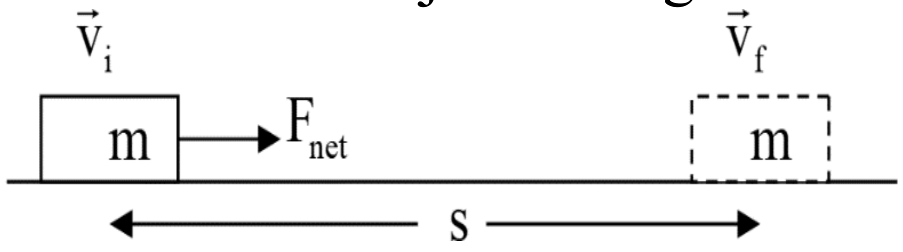
mass
kg

Velocity
m/s

Work and Energy: Kinetic Energy

Kinetic Energy :

Let us now consider an object with a mass m and initial velocity v_i (as in the adjacent figure), so its initial kinetic energy is K_i , if we want to accelerate it so that its final velocity is v_f (and therefore its final kinetic energy is K_f), then a constant force must be applied for a distance s, so the work done on the object during that distance is:



$$W_{net} = F_{net}\, s$$

This work must be equal to the change in kinetic energy during that distance,

$$W_{net} = K_f - K_i$$

$$W_{net} = F_{net}\, s = K_f - K_i$$

$$= F_{net}\, s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Work and Energy: Example 6

Example 6:

How much work is needed to accelerate a car of mass 1000 kg from 20 m/s to 30 m/s?

Solution

$$w = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}(1000)(30)^2 - \frac{1}{2}(1000)(20)^2$$

$$= 450{,}000 - 200{,}000$$

$$= 250{,}000\ J$$

$$= 250\ \text{kJ}$$

Work and Energy: Potential energy

Potential energy:

It is the energy stored in a system (a body) due to a work done on it, which leads to a change in its position (as in the case of work done to lift an object against the force of gravity) or a change in its shape and dimensions (as in the case of work done on a spring). We will focus in our study on the potential energy resulting from changing the position of the body.

Suppose an object of mass m located at a height h from the ground, then it has a potential energy U with respect to the ground given by:

$$U = mgh$$

Total mechanical energy (E): represents the sum of the kinetic energy and potential energy of the object, that is:

$$E = K + U$$

Work and Energy: Energy conservation principle

Energy conservation principle :

This principle states that the total energy of an isolated system always remains constant, i.e.:

$$E = K + U = \text{constant}$$

if E_i and E_f are total initial and final energy respectively, then :

$$E_i = E_f = \text{constant}$$

$$K_i + U_i = K_f + U_f = \text{constant}$$

where: K_i and K_f are the initial and final kinetic energy, respectively. U_i and U_f are the initial and final potential energy, respectively.

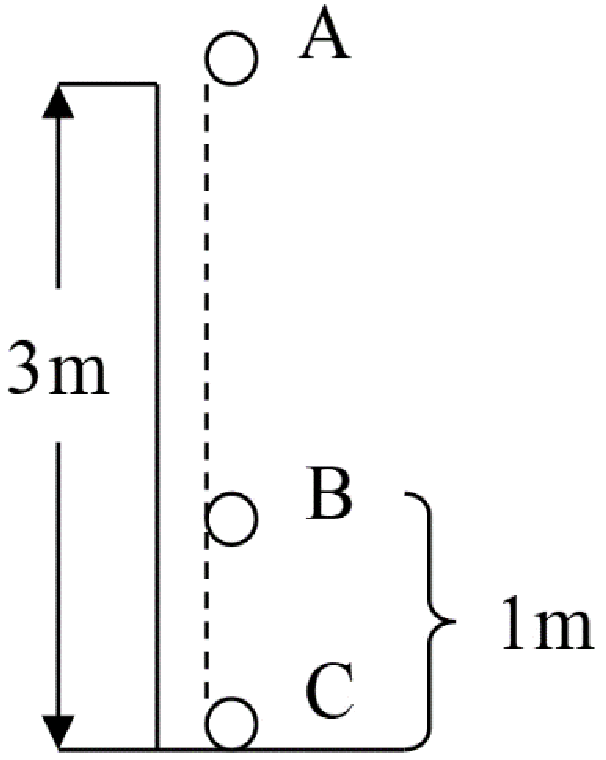
Work and Energy: Example 7

Example 7:

A ball is in free fall from a height of 3 m (as shown in the figure below). Calculate the speed of the ball:

(a) At a height of 1 m from the ground.

(b) When it hits the ground



Solution

(a) The total energy is constant, i.e.:

$E_A = E_B = E_C$

$K_A + U_A = K_B + U_B = \text{constant}$

$\frac{1}{2}mv_A^2 + mgh_1 = \frac{1}{2}mv_B^2 + mgh_2$

Since $v_A = 0$: , canceling m from both sides we get:

$\frac{1}{2}v_B^2 = g(h_1 - h_2)$

$\therefore v_B = \sqrt{2g(h_1 - h_2)} = \sqrt{2(9.8)(3-1)} = 6.3\ m/s$

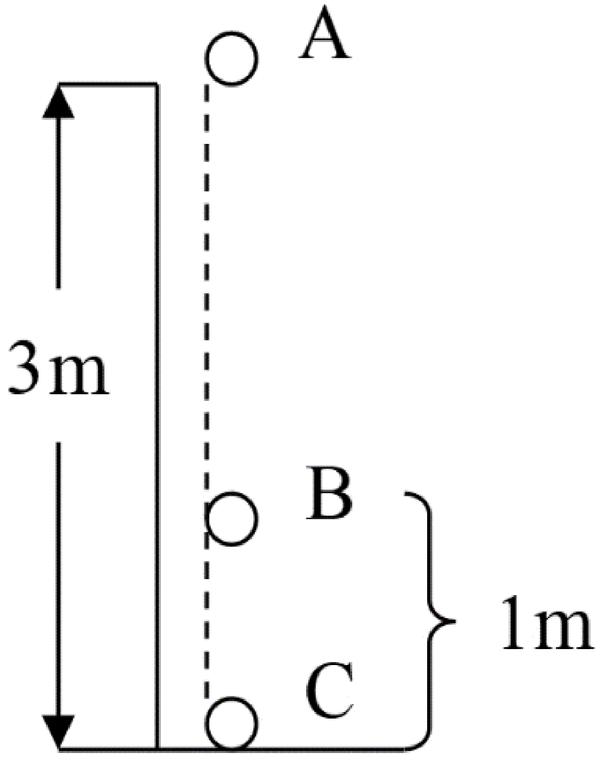
Work and Energy: Example 7 (continued)

Example 7:

A ball is in free fall from a height of 3 m (as shown in the figure below). Calculate the speed of the ball:

(a) At a height of 1 m from the ground.

(b) When it hits the ground



Solution

(b) $E_A = E_C$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_C^2 + mgh_B$$

$$\Longrightarrow 0 + mgh_A = \frac{1}{2}mv_C^2 + mgh_B$$

$$\Longrightarrow \frac{1}{2}v_C^2 = g(h_A - h_C)$$

$$\therefore v_C = \sqrt{2g(h_A - h_C)}$$

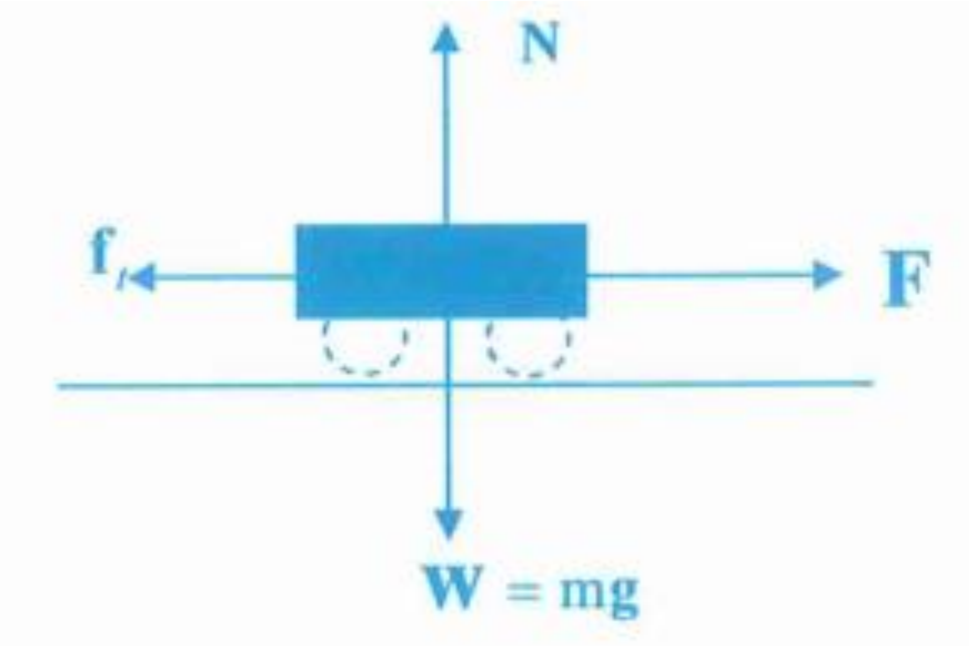
$$= \sqrt{2(9.8)(3-0)} = 7.7\ m/s$$

Work and Energy: Conservation of Energy in the presence of Friction

Energy conservation principle :

In the case of friction, the loss in mechanical energy = the work caused by friction:

$$E_i - E_f = W_f$$

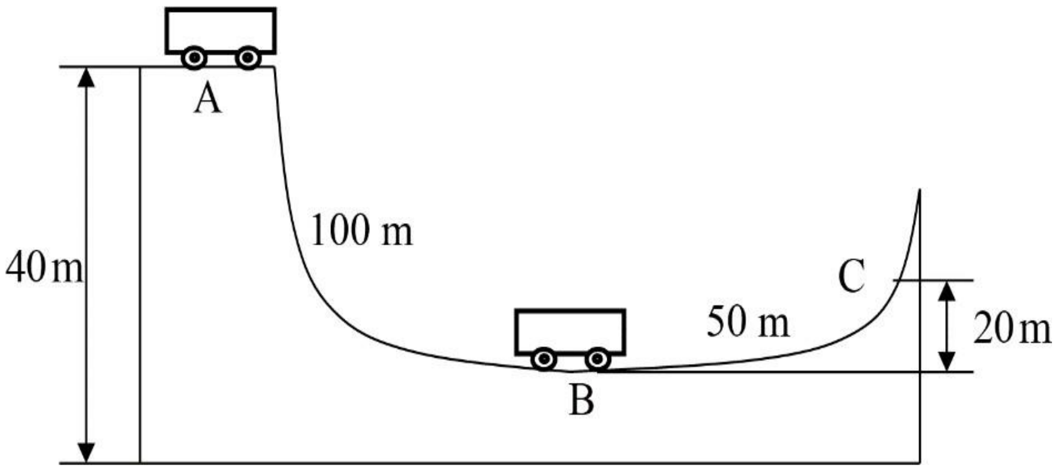


Work and Energy: Example 8

Example 8:

Looking at the path shown in the adjacent figure, the cart is moving from rest at point A, calculate the speed of the cart at the bottom of the hill (point B) given that the surface is smooth (frictionless).

Solution



$$E_A = E_B$$

$$\because \frac{1}{2}mv_A^2 + mgh_1 = \frac{1}{2}mv_B^2 + mgh_2$$

$$\Longrightarrow 0 + mgh_1 = \frac{1}{2}mv_B^2 + 0$$

$$\Longrightarrow mgh_1 = \frac{1}{2}mv_B^2$$

$$\Longrightarrow v_B^2 = 2gh_1$$

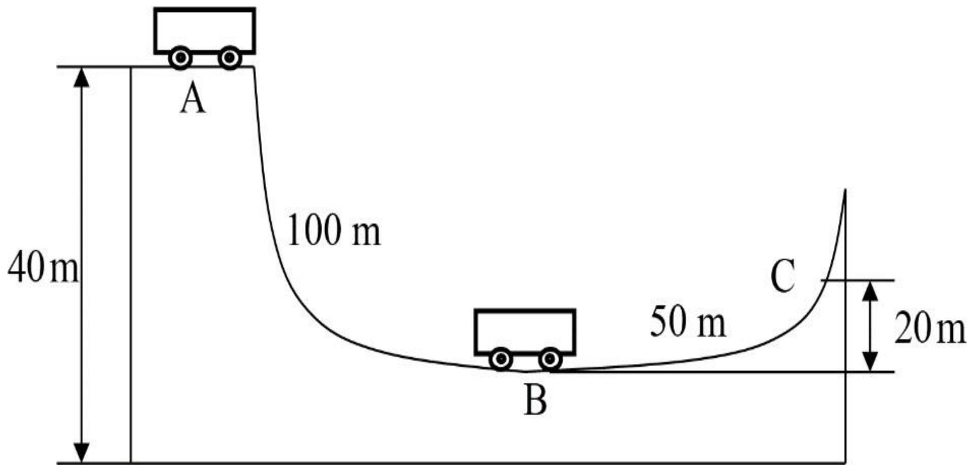
$$\therefore \quad v_B = \sqrt{2gh_1} = \sqrt{2(9.8)(40)} = 28\, m/s$$

Work and Energy: Example 9

Example 9:

If we consider the surface on which the cart is traveling in the previous example has a constant friction force of 6 N, and the mass of the cart is 30 kg, (a) what is the cart's velocity at the bottom of the hill (point B)? (b) What is its velocity at point C?

Solution



$(a)\quad E_A - E_B = W_f$

$$\frac{1}{2}mv_A^2 + mgh_A - \left(\frac{1}{2}mv_B^2 + mgh_B\right) = 6 \times 100$$

$$0 + mgh_A - \left(\frac{1}{2}mv_B^2 + 0\right) = 6 \times 100$$

$$v_B^2 = 2\left(\frac{mgh_A - 600}{m}\right) = 2\left(\frac{30 \times 9.8 \times 40 - 600}{30}\right)$$

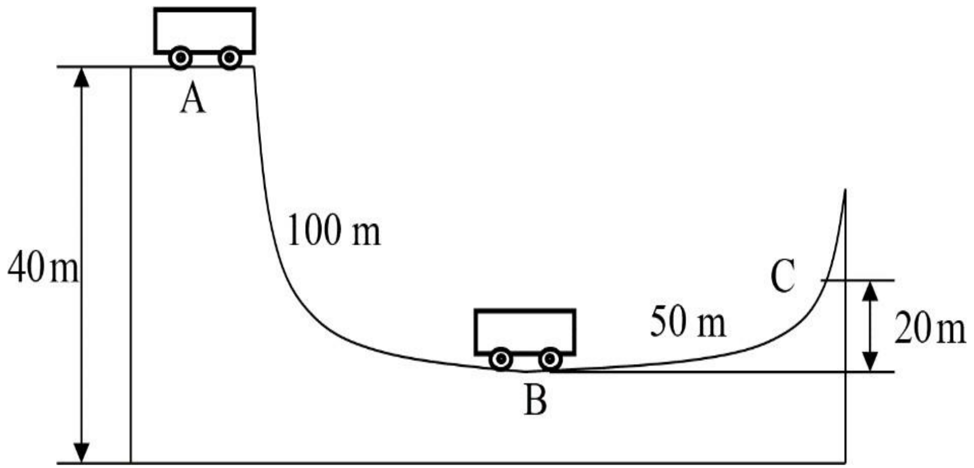
$$v_B = \sqrt{2\left(\frac{30 \times 9.8 \times 40 - 600}{30}\right)} = 27.3\ m/s$$

Work and Energy: Example 9

Example 9:

If we consider the surface on which the cart is traveling in the previous example has a constant friction force of 6 N, and the mass of the cart is 30 kg, (a) what is the cart's velocity at the bottom of the hill (point B)? (b) What is its velocity at point C?

Solution



$$(b)\quad E_A - E_C = W_f$$

$$\frac{1}{2}mv_A^2 + mgh_A - \left(\frac{1}{2}mv_C^2 + mgh_C\right) = 6 \times 50$$

$$0 + mgh_A - \left(\frac{1}{2}mv_C^2 + mgh_C\right) = 300$$

$$v_C^2 = 2\left(\frac{mgh_A - 300 - mgh_C}{m}\right) = 2\left(\frac{30\times9.8\times40 - 300 - 30\times9.8\times20}{30}\right)$$

$$v_C = \sqrt{2\left(\frac{30 \times 9.8 \times 40 - 300 - 30 \times 9.8 \times 20}{30}\right)} = 19.3\, m/s$$

Thanks

King Saud University

College of Science

Phys 105

Physics for Architecture Students

Material properties

Chapter 12

Elasticity

Material properties : Density (d)

Density:

Some material properties like stress and strain determine how much load a material can withstand before permanent deformation occurs. starting with some basic definitions:

Density (d): The density of a substance is defined as the product of its mass divided by its volume, that is:

$$d = \frac{m}{V}$$

where m is the mass of the substance, V: material volume.

The units of density is kg/m^3 according to SI system

Material properties : Example 1

Example 1:

The density of aluminum is 2.7 g/cm^3, what is its density in SI unit system?

Solution:

$$d = 2.7\frac{g}{cm^3} \times \frac{10^{-3}\frac{kg}{g}}{10^{-6}\frac{m^3}{cm^3}}$$

$$= 2700 \text{ kg}/m^3$$

Material properties : Example 2

Example 2:

What is the mass and weight of the air in a square room of 4 m side and 3 m height if the density of air at sea level is 1.28 kg/ m^3

Solution:

$$V = 4 \times 4 \times 3 = 48\, m^3$$

$$\because \quad d = \frac{m}{V}$$

$$\therefore \quad m = Vd$$

$$= 48 \times 1.28 = 61.44\, kg$$

$$\Longrightarrow w = mg = 61.44 \times 9.8 = 602\, N$$

Material properties : Stress(σ)

Stress (σ):

It is the force applied per unit area of the surface to which the force is applied.

Stress = σ = force/Area = F/A

Unit of stress is: Pa

$$1Pa = \frac{1N}{m^2}$$

Material properties : Longitudinal Strain (ε)

Longitudinal Strain (ε):

It is defined as: Change in the length to the original length of an object. It is caused due to longitudinal stress and is denoted by the Greek letter epsilon $\boldsymbol{\varepsilon}$.

$$\text{Strain } \varepsilon = \frac{\Delta L}{L}$$

Where ΔL is the change in length, L is the original length

The strain is *dimensionless*

Material properties : Modulus of elasticity (E)

Modulus of elasticity (E):

The modulus of elasticity is the ratio between stress (compression, tensile, or shear) to strain , and it is given as:

Modulus of elasticity(E): Stress/Strain ; its unit is **Pa.**

$$\mathrm{E} = \frac{\sigma}{\varepsilon}$$

$$\mathrm{E} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

Material properties : Young's modulus (Y)

Young's modulus (Y):

If the stress is the result of a tensile or compressive force, then the modulus of elasticity is called Young's modulus (Y), meaning that:

Young's modulus (Y) = Longitudinal Stress/ Longitudinal Strain

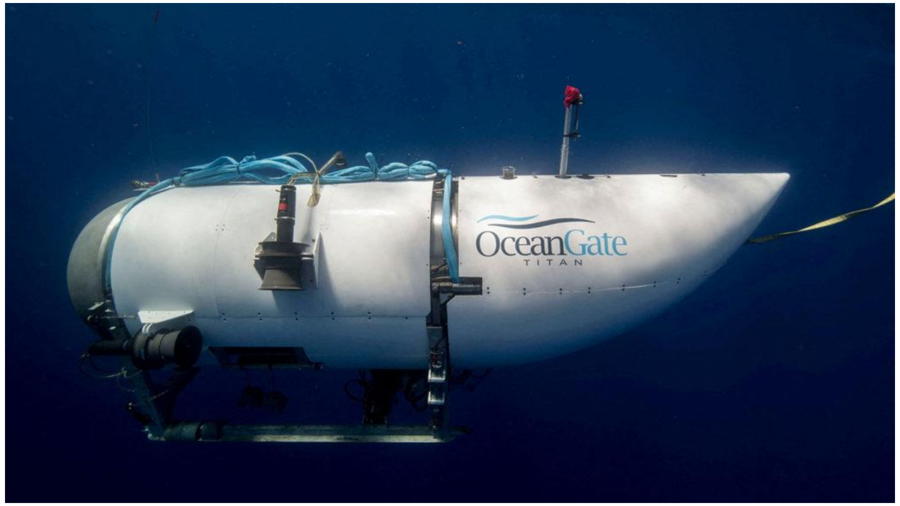
$$\mathrm{Y} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

The Young's modulus depends on the type of body material and does not depend on its shape or size.

Material properties : Bulk modulus (B)

Bulk modulus (B) :

When a compressive force acts on all surfaces of an object, its volume decreases. If the applied force per unit area F/A is uniform, then the stress in this case is the applied pressure P, so we define the Bulk modulus:



Bulk modulus (B)= Volume Stress/Volume Strain

$$B = \frac{\frac{F}{A}}{\frac{\Delta V}{V_o}} = \frac{P}{\frac{\Delta V}{V_o}} = \frac{PV_o}{\Delta V}$$

Where V_o :initial volume

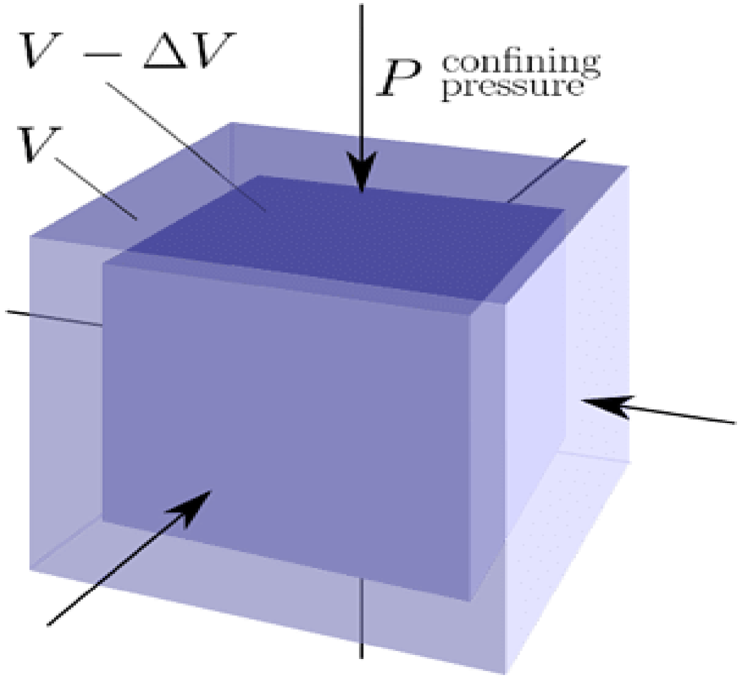
ΔV :change in volume

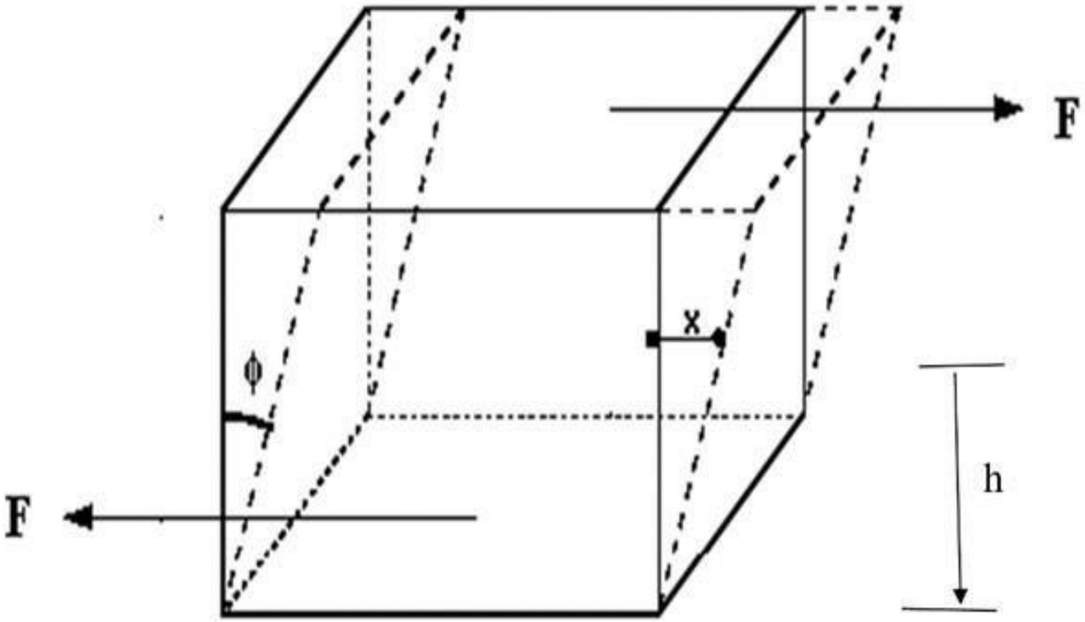
The Bulk modulus is a measure of the degree of difficulty with which a material is compressed. The unit of Bulk modulus is the pascal (Pa).

Material properties : Shear Stress (σ_s)

Shear Stress (σ_s):

We suppose a parallelepiped shape whose area is A and is subjected to a shear force F within the elasticity, as shown in the figure shown:





Material properties : Shear Stress (σ_s)

Shear Stress (σ_s):

The shear force F leads to a change in its shape to become as in the diagram, and thus we know the shear stress:

$$\sigma_s = \frac{F}{A}$$

We define Shear strain $\varepsilon_s = x/h$ Where x and h are shown on the diagram ,then we can write :

$\tan\varphi = x/h$

when $x \ll h$, then we can write :

$\tan\varphi \approx \varphi = x/h = \varepsilon_s$

We define the shear modulus G as :

$$G = \frac{F/A}{x/h} = \frac{F}{A\varphi}$$

Material properties : Hooke's Law

Hooke's Law:

Within the limits of elasticity of the spring, the restoring force of the spring F to its original position is directly proportional to the displacement of its pull or compression x .

The restoring force of the spring F = - spring constant x displacement

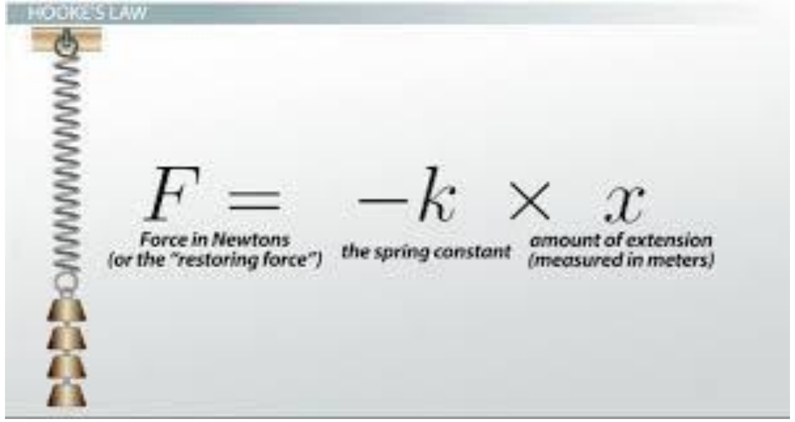
$$F = -kx$$

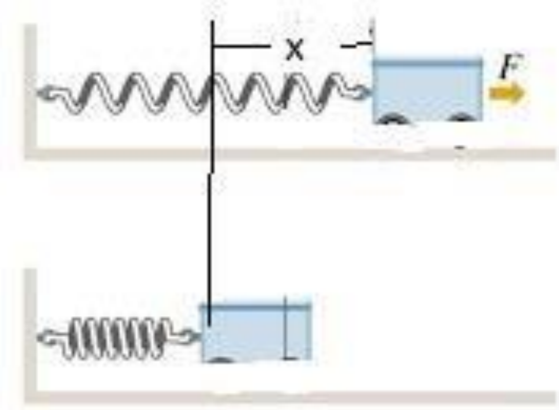
k is the spring constant
x is the change in length

Material properties : Hooke's Law

Hooke's Law:

The spring force is called a restoring force because **the force exerted by the spring is always in the opposite direction to the displacement**. This is why there is a negative sign in the Hooke's law equation





Hooke's law can be written as:
Stress = Young's modulus x strain

$$\sigma = Y\varepsilon$$

Material properties : Example 3

Example 3:

A weight of 45 N was attached to a spring. When measuring the length of the spring, it was found that it was 32 cm. The 45 N weight was removed and another weight was attached with W = 55, so the spring is elongated by 13 cm. Calculate:

a) Spring constant

b) The original length of the spring

Solution:

a) $F = -kx$

Applying Hooks law on the second weight :

$$k = \frac{F_2}{x_2} = \frac{55}{0.13} = 429 \ \ N/m$$

b) Applying Hooks law on the second weight :

$$x_1 = \frac{F}{k} = \frac{45}{429} = 0.105\, m = 10.5\, cm \Longrightarrow L = 32 - 10.5 = 21.4\, cm$$

Material properties : Example 4

Example 4:

A wire of length 2.5 m with a cross-sectional area $6 \times 10^{-6}\ m^2$ with a 45-kg mass attached to its end gives an elongation of 1.27 mm.

Calculate: (a) Stress on the wire. (b) Strain. (c) Young's modulus of wire material.

Solution:

(a) Stress: $\sigma = \frac{mg}{A} = \frac{45 \times 9.8}{6 \times 10^{-6}} = 73.5 \times 10^{7}$ Pa

(b) Strain: $\varepsilon = \frac{\Delta L}{L} = \frac{1.27 \times 10^{-3}}{2.5} = 5.08 \times 10^{-4}$

(c) $Y = \frac{\sigma}{\varepsilon} = \frac{73.5 \times 10^{7}}{5.08 \times 10^{-4}} = 1.44 \times 10^{11}$ Pa

Material properties : Example 5

Example 5:

An aluminum wire of 3 mm diameter and 4 m length is used to support a mass of 50 kg. What is the elongation of the wire? (Young modulus for aluminum is: 7×10^{10}).

Solution:

$$A = \pi r^2 = (3.14)\left(\frac{3}{2} \times 10^{-3}\right)^2 = 7.07 \times 10^{-6}\ \mathrm{m}^2$$

$$F = mg = 50 \times 9.8 = 490\ \mathrm{N}$$

$$\mathrm{Y} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$\Longrightarrow \Delta L = \frac{L}{Y} \times \frac{F}{A} = \frac{4}{7\times10^{10}}\frac{490}{7.07\times10^{-6}} = 3.96 \times 10^{-3}\ \mathrm{m}$$

$$\Delta L = 3.96\, mm$$

Example 6:

A pressure of 2×10^6 Pa is applied to a sample of mercury and its volume decreases by an amount of 0.008% . Calculate the Bulk modulus of mercury.

Solution:

$$\mathrm{B} = \frac{\frac{F}{A}}{\frac{\Delta V}{V_O}} = \frac{P}{\frac{\Delta V}{V_O}} = \frac{(2\times10^6)(100)}{0.008}$$

$$= 2.5 \times 10^{10}\ \mathrm{Pa}$$

Material properties : Pressure (P)

Pressure (P):

Fluid is a substance which can flow and it includes both gases and liquids. Since a fluid has no definite shape ,it can take the shape of its containers, e.g. pipes ,channels, arteries ,etc. It is convenient to use pressure when dealing with fluids (rather than use force) because the force they exert on their containers is uniformly distributed over the area.

Pressure is defined as : the perpendicular force acting on the unit area of the surface, and is given by:

$$\mathrm{P} = \frac{F}{A}$$

where F is the vertical force, A is the surface area.

Pressure is measured in pascals (Pa), where:1 Pa =1 N/m^2

Material properties : Example 7

Example 7:

(a) A person weighing 500 N is standing on the ice of a frozen pond with his feet against an area of ice. What is the pressure on the ice?

(b) If you know that the ice will collapse at a pressure of 16,000 Pa, how much weight would a person need to make the ice collapse, given the same contact area as before?

Solution:

(a) $\mathrm{P} = \frac{F}{A} = \frac{500}{0.05} = 10\ \mathrm{kPa}$

(b) $\mathrm{F} = PA = 16{,}000 \times 0.05 = 800\ \mathrm{N}$

Thanks

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Material properties

Chapter 13 and 14

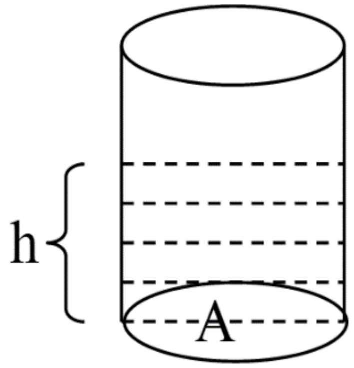
Fluids

Static Fluid : Pressure increases with depth

Pressure increases with depth:

To calculate the pressure applied by a column of liquid of length h on the bottom area A in the adjacent figure, we apply the relationship:

$$P = \frac{F}{A}$$



where F is the force perpendicular to the area A, which is equal to the weight of the column of liquid, that is:

$$\mathrm{F = mg = Vdg \; F = hAdg}$$

where V: volume of the liquid column, d : density of the liquid.

$$P = \frac{\mathrm{hAdg}}{A} = \mathrm{hdg}$$

Static Fluid : Pressure increases with depth

The relationship shows an increase in pressure with an increase in depth and with an increase in the density of the liquid.

And if we want to calculate the total pressure P_t affecting the bottom area A, we add the atmospheric pressure P_0 (because the container is open) to the pressure of the liquid column, that is:

$$P_t = P_0 + P$$

$$\rightarrow P_t = P_0 + hdg$$

It is known that the atmospheric pressure at sea level is equal to:

$$P_0 = 1.013 \times 10^5 \text{ Pa}$$

and this represents the pressure of a column of mercury with a height of 76 cm, as shown in the example:

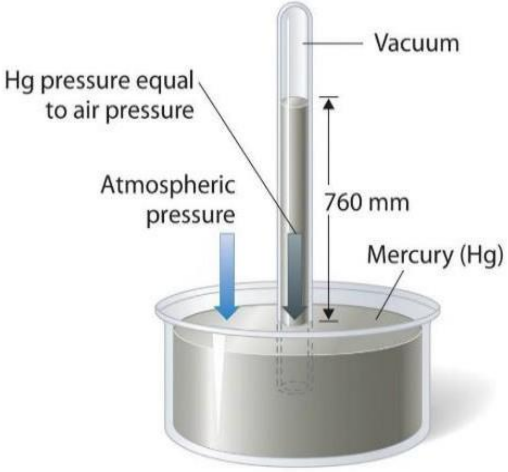
Pressure increases with depth : Example 1

Example 1:

Find the pressure of a column of mercury with a height of 76 cm, knowing that the density of mercury.

Solution:

$$P = \text{hdg}$$
$$= 0.76 \times 13600 \times 9.8$$
$$= 1.01 \times 10^5\ Pa = 1\ atm$$



Mercury Barometer

So, 76 cm of mercury causes a pressure of 1 atm

Pressure increases with depth : Example 2

Example 2:

What is the total pressure at the bottom of a 2 m deep swimming pool completely filled with water?

Solution:

$$P_t = P_0 + hdg$$
$$= 1.013 \times 10^5 + 2 \times 1000 \times 9.8$$
$$= 1.013 \times 10^5 + 19600$$
$$= 1209600\ Pa$$
$$= 1.21 \times 10^6\ Pa$$

Static Fluid : Pascal´s Principal

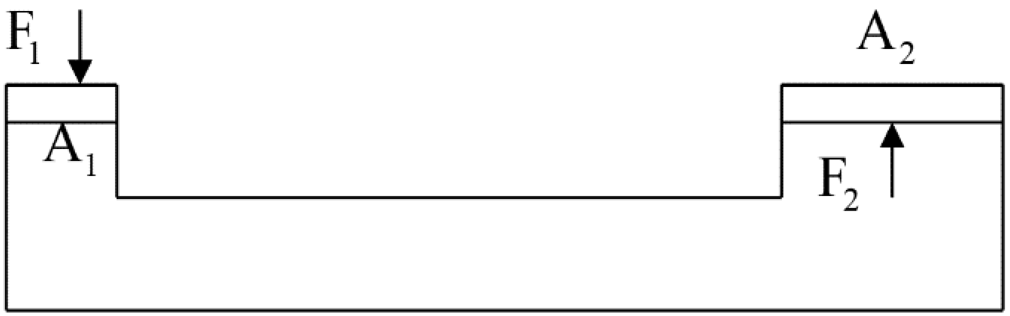
Given that the pressure is equal at all points of equal depth in the liquid, any increase in pressure on the surface must be transmitted to any point in the liquid equally, which is called Pascal's law, which states the following:

The external pressure applied to a liquid within a closed vessel is transmitted without any decrease to all points of the liquid and to the walls of the closed vessel.

As an application on Pascal's Principal we have the hydraulic jack shown in the adjacent figure,:

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



Pressure increases with depth : Example 3

Example 3:

A barber chair is placed on a hydraulic piston with a diameter of 10 cm, while the area of the lifting piston is 10 cm^2. If the mass of the chair and the seated person is 160 kg, what force must be applied to lift the piston?

Solution:

$$A_1 = 10\ cm^2 = 0.001 m^2; F_1 = ?$$
$$r_2 = 5\ cm = 0.05\ m \rightarrow A_2 = \pi r_2^2 = 3.14 \times (0.05)^2 = 7.85 \times 10^{-3} m^2$$
$$F_2 = m_2 g = 160 \times 9.8 = 1568\ \text{N}$$
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$
$$F_1 = F_2(A_1/A_2) = 1568(0.001/7.85 \times 10^{-3})$$
$$F_1 = 200\ N$$

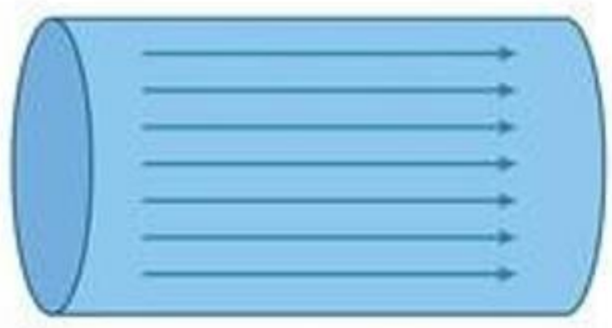
Fluids : Fluid Dynamics

Fluid dynamics describes the flow of fluids — liquids and gases. In our study of fluid flow, we will assume that it is an ideal flow, assuming the following:

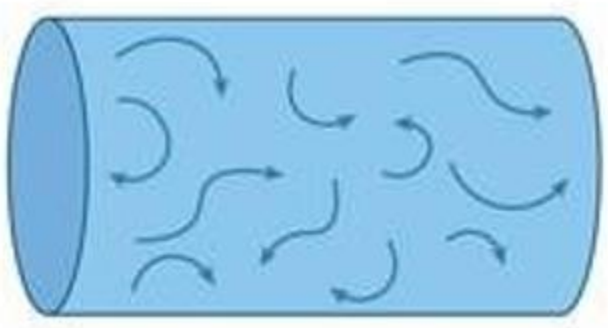
1- *Incompressible*, meaning that the density of the fluid is constant.

2- *Not viscous*, meaning we neglect the friction between the fluid layers.

3- *The flow of the fluid is streamlined*, i.e. the velocity of a specific point of the fluid does not change with time.



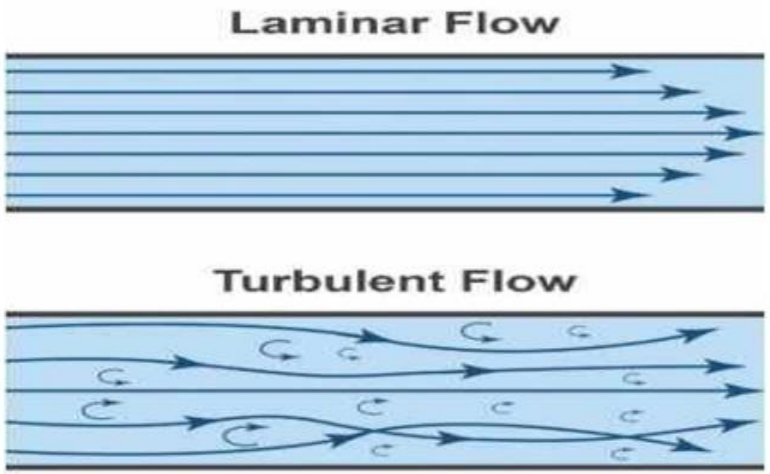
Streamline Flow



Turbulent Flow

Fluids : Equation of Continuity

A streamline is the path followed by a fluid particle in a steady flow. In streamline (laminar) flow, the fluid moves smoothly in parallel layers, and the streamlines are continuous and never intersect, as shown in the figure.



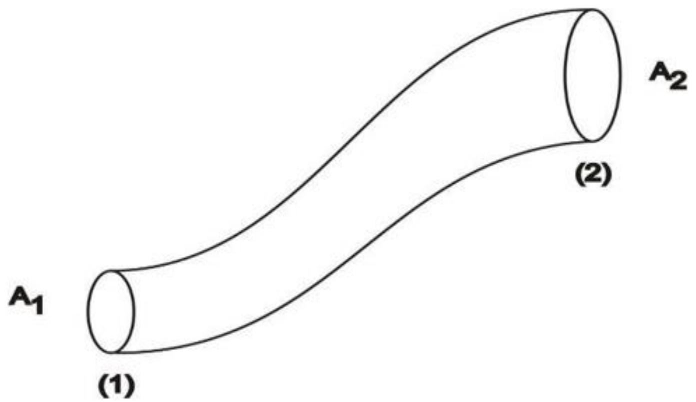
In contrast, in turbulent flow, the fluid motion is irregular, with eddies and vortices. In this case, streamlines lose their smooth character, may swirl, and can even locally reverse direction.

Turbulent flow typically occurs at high velocities or when the fluid encounters obstacles or sharp edges.

The continuity equation is derived for streamline (steady, non-turbulent) flow and expresses the conservation of mass of the fluid as it moves through a pipe or channel of varying cross-section.

Fluids: Equation of Continuity

Consider the tube shown in the figure, through which a fluid flows in the direction shown. Since the rate of mass passing through any section of the pipe is a conserved quantity, so the rate of mass passing through section (1), is equal to the rate of mass passing through section (2).



mass flow rate = $d_1 A_1 v_1 = d_2 A_2 v_2 = \text{constant}$

Where:

d_1 , d_2 : , are the density at sections (1) and (2), respectively.

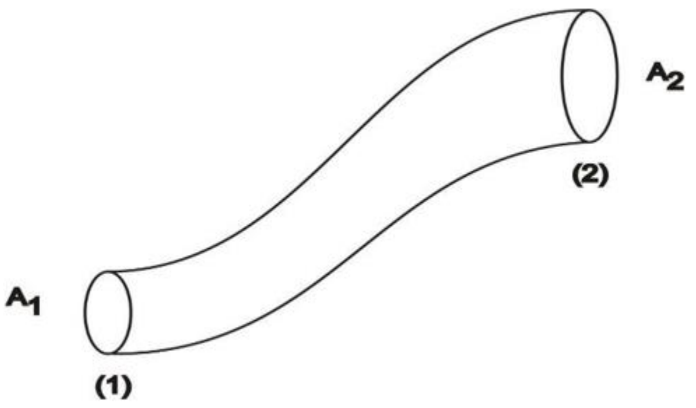
v_1 , v_2 represents the velocity at segments (1) and (2), respectively.

A_1 , A_2 are the cross-sectional areas at sections (1) and (2), respectively

Fluids : Equation of Continuity

For incompressible liquids such as water, for example, the density value is constant, and therefore the volumetric flow rate Q is constant, meaning that:

Continuity equation for incompressible liquids



volumetric flow rate Q = $A_1 v_1 = A_2 v_2 = \text{constant}$

The volumetric flow rate is measured by the SI units: m^3/s

Equation of Continuity: Example 4

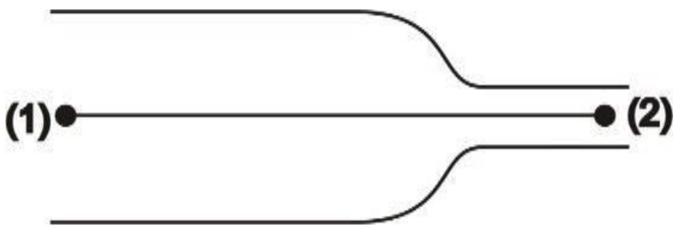
Example 4:

A tube in which water flows as shown in the figure, so that: v_1=1.8 m/s, r_1 = 12.5 mm, r_2 = 9 mm, calculate:

a. the velocity at the narrow part of r.

b. What is the volumetric flow rate?

c. What is the mass flow rate?



Equation of Continuity: Example 4

Solution:

a.

$A_1 v_1 = A_2 v_2$

$$v_2 = \frac{A_1 v_1}{A_2} = v_1 \frac{\pi r_1^2}{\pi r_2^2} = v_1 \left(\frac{12.5}{9}\right)^2 = 3.5\ m/s$$

b. the volumetric flow rate : Q = $A_1 v_1 = A_2 v_2$

$\Longrightarrow$ Q = $3.14 \times (12.5 \times 10^{-3})^2 \times 1.8$

$\Longrightarrow$ Q = $8.8 \times 10^{-4}\ m^3/s$

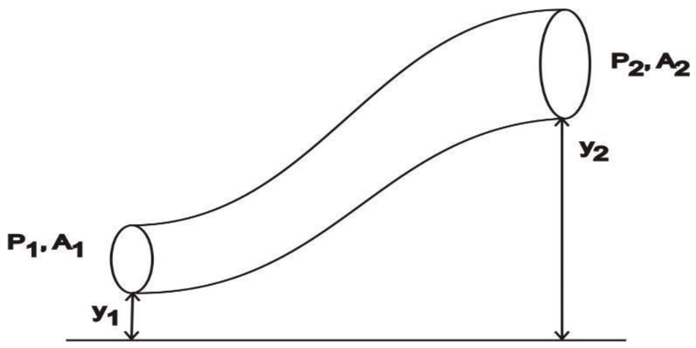
c. mass flow rate = $d_1 A_1 v_1 = d_2 A_2 v_2$

$= 1 \times 10^3 \times 8.8 \times 10^{-4}$

$= 0.88\ kg/s$

Fluids : Bernoulli ´s equation

Bernoulli was able in 1738 to prove that the pressure is inversely proportional to the velocity of the fluid for the streamline flow. The Bernoulli equation can be thought of as a relationship for the conservation of energy during a given fixed volume of liquid, which can be written by referring to the adjacent figure by the form:



$$P_1 + \frac{1}{2} d v_1^2 + d g y_1 = P_2 + \frac{1}{2} d v_2^2 + d g y_2 = \text{constant}$$

This means that: the sum of pressure, kinetic energy per unit volume and potential energy per unit volume is a constant quantity at all points on the fluid streamline flow.

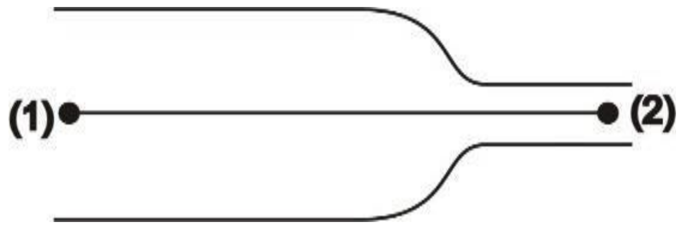
As a special case of the Bernoulli equation when the fluid is at rest, we have: $v_1 = v_2 = 0$

$$P_1 - P_2 = dg(y_2 - y_1)$$

Equation of Continuity: Example 5

Example 5:

For the previous example, if the pressure at position (1) is (51 kPa), what is the pressure at position (2)?



Solution:

Since height $y_1 = y_2$, as well as density $d_1 = d_2 = d$, then Bernoulli's equation is:

$$P_1 + \frac{1}{2} d v_1^2 = P_2 + \frac{1}{2} d v_2^2 = \text{constant}$$

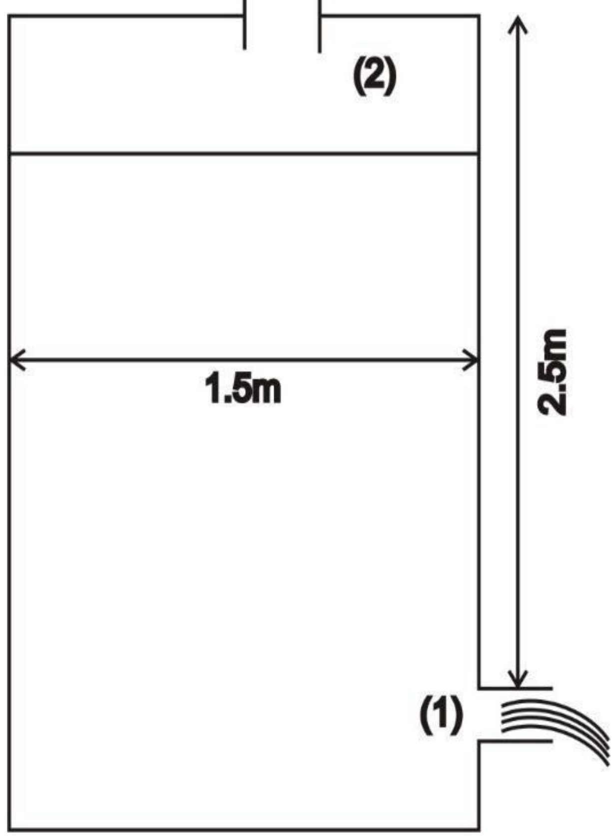
$$P_2 = P_1 + \frac{1}{2} d (v_1^2 - v_2^2)$$

$$= 5.1 \times 10^3 + \frac{1}{2} \times 10^3 [(1.8)^2 - (3.5)^2] = 4.7 \times 10^4 \ Pa$$

Equation of Continuity: Example 6

Example 6:

A water tank with an inner diameter of (1.5 m) and a hole with a diameter of (15 mm) on the side and a distance of (2.5 m) from the height of the water in the tank (see figure). What is the velocity of the water leaving the hole?



Solution:

$$P_1 + \frac{1}{2} d v_1^2 + d g y_1 = P_2 + \frac{1}{2} d v_2^2 + d g y_2$$

The pressure at positions (1) and (2) is the same, equal to atmospheric pressure. That is $P_1 = P_2$ .Since the density is the same, then the Bernoulli equation is in the form:

$$\frac{1}{2} v_1^2 + g y_1 = \frac{1}{2} v_2^2 + g y_2 \qquad (1)$$

Equation of Continuity: Example 6

The pressure at positions (1) and (2) is the same, equal to atmospheric pressure. That is $P_1 = P_2$ .Since the density is the same, then the Bernoulli equation is in the form:

$$v_1 A_1 = v_2 A_2 \qquad (2)$$

$$v_2 = \frac{A_1}{A_2} v_1$$

$$= \left(\frac{0.015}{1.5}\right)^2 v_1$$

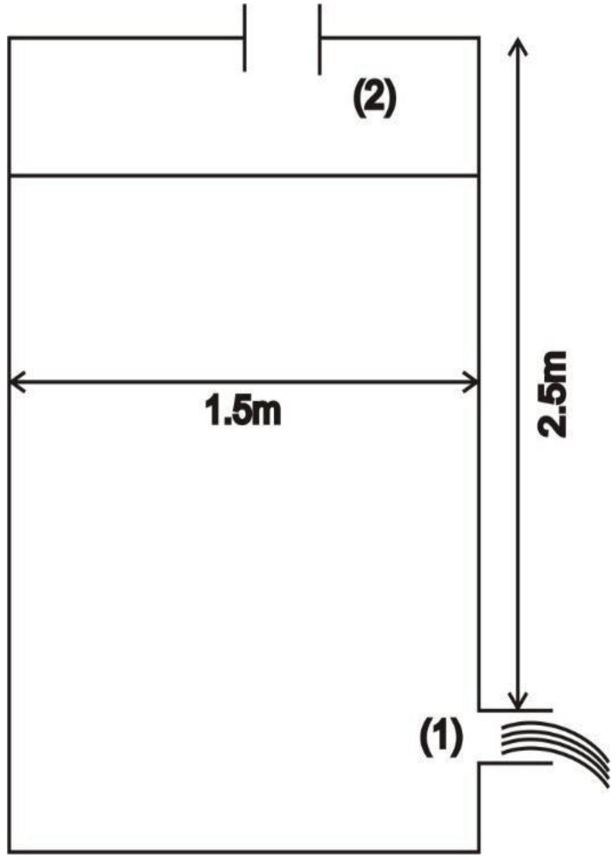
$$\therefore v_2 = 0.0001 v_1 \approx 0$$

Substituting into the equation (1):

$$v_1^2 = 2g(y_2 - y_1) = 2 \times 9.8 \times 2.5$$

$$\therefore v_1^2 = 49$$

$$\therefore v_1 = 7\ m/s$$



Thanks

King Saud University

College of Science

Phys 105

Physics for Architecture Students

Heat

Chapters 17 + 18

Heat

Heat: Heat

Heat:

Heat is energy in transit that flows between a system (such as a body) and its surroundings due to a temperature difference. Heat always transfers from the higher-temperature body to the lower-temperature one until thermal equilibrium is reached.

- In the International System of Units (SI), heat is measured in **joules (J)**.
- Historically, heat was measured in **calories (cal)**, where: **1 cal =4.186 J** . This constant is known as **Joule's mechanical equivalent of heat**, established by James Joule, who demonstrated that mechanical work and heat are equivalent forms of energy.

Heat: temperature

Temperature:

Temperature is a **measure of the average kinetic energy** of the microscopic particles (atoms or molecules) of a system. It determines the direction of heat flow between two bodies in thermal contact.

- SI unit: **Kelvin (K)**.
- Other scales: **Celsius (°C)** and **Fahrenheit (°F)**.
- Example: Raising the temperature of **1 g of water by 1°C** requires approximately **4.186 J** of energy.

Heat: Thermal Energy (Internal Energy)

Thermal Energy (Internal Energy):

Thermal energy, often referred to as **internal energy**, is the **total microscopic kinetic and potential energy** of all particles within a system. Unlike heat, which is energy in transfer,

internal energy is a **state property** of the system.

- It depends on factors such as temperature, the number of particles, and the type of substance.
- Measured in **joules (J)**.

Heat: Thermometers

Thermometers:

The most famous thermometers are the mercury thermometer and the electrical resistance thermometer.

The scales used in thermometers are of three well-known types, namely:

1- Celsius scale

2- Fahrenheit scale

3- **The absolute scale**, which is used in the International System of Units

Heat: Thermometers

Thermometers:

The relationships between the three scales are:

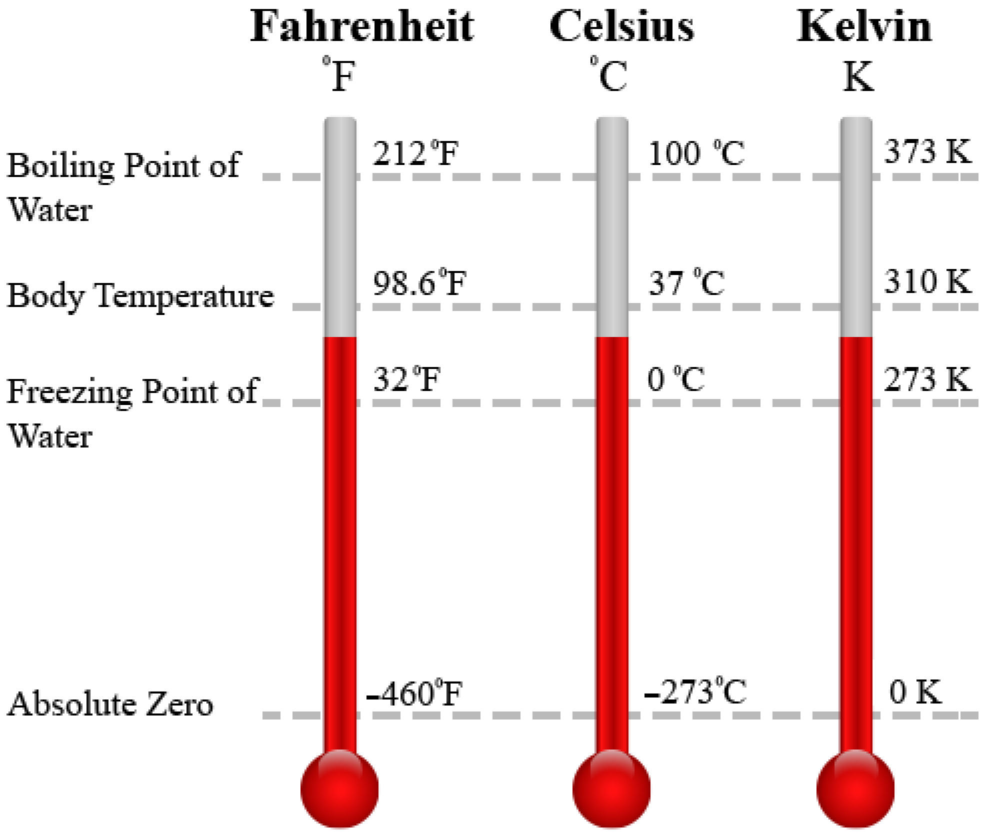
$$T_c = \frac{5}{9}(T_F - 32), \; T_F = \frac{9}{5}T_c + 32$$

$$T_c = T_K - 273, \quad T_K = T_c + 273$$

$$T_F = \frac{9}{5}(T_K - 273) + 32$$

$$T_K = \frac{5}{9}(T_F - 32) + 273$$

Where T_K , T_F , T_C correspond to temperatures in absolute, Fahrenheit, and Celsius scales, respectively. The following is a comparison of temperatures in the three scales :



Heat: Example 1

Example 1:

If the room temperature is $(77^{o}F)$, what is the degree in Celsius?

Solution:

$$T_c = \frac{5}{9}(T_F - 32)$$

$$= \frac{5}{9}(77 - 32) = 25^{o}C$$

Heat: Example 2

Example 2:

What is the temperature on the Fahrenheit scale on a day when the call temperature on the Celsius scales is $-10^{o}C$?

Solution:

$$T_F = \frac{9}{5}T_c + 32$$

$$= \frac{9}{5}(-10) + 32 = 14^{o}F$$

Heat: Example 3

Example 3:

If the temperature is on the Celsius scale $-70^o C$, what is the temperature on the absolute scale?

Solution:

$$T_K = T_c + 273$$
$$= -70 + 273 = 203\ K$$

Heat: Specific Heat

Specific Heat:

It is the amount of heat required to raise the temperature of a unit mass of a substance one degree Celsius (or absolute).

Specific heat unit is: $\frac{cal}{g.{}^{o}C}$ or in SI system: $\frac{J}{kg.K}$

For example, the specific heat of water is: $c = 4186\frac{J}{kg.K} = 1\ \frac{cal}{g.{}^{o}C}$.

This means that a mass of water of 1g needs to gain 1 cal of heat to raise its temperature by one degree Celsius. Or that 1 kg of water needs to gain a heat of 4186 J in order to increase its temperature by one absolute degree.

The specific heat of iron is:

$c = 450\frac{J}{kg.K}$. for sand, $c = 820\frac{J}{kg.K}$, and for wood $c = 1680\frac{J}{kg.K}$

Heat: Specific Heat

Specific Heat:

Using the above definition, we can calculate the heat needed to raise the temperature of a mass of material (m) its by an amount ΔT from the following relationship:

$$\Delta Q = mc\Delta T$$

where c is the specific heat capacity of the substance, ΔT is the difference between the initial and final temperature.

Heat: Example 4

Example 4:

What is the heat required to heat 20 g of water from 30^oC to 90^oC?

Solution:

the specific heat of water is: $c = 1\ \frac{cal}{g.^oC}$

$$\Delta Q = mc\Delta T$$
$$= 20 \times 1 \times (90 - 60)$$
$$= 1200\ cal.$$
$$\because 1\ cal = 4.186\ J$$
$$\therefore \quad \Delta Q = 1200 \times 4.186 = 5023\ J$$

Heat: Example 5

Example 5:

A child has a mass of 30 kg and his temperature is 39^oC . How much heat must be removed from the child's body so that his temperature becomes 37^oC , knowing that the specific heat of the human body $3470\,\frac{J}{kg.K}$?

Solution:

$$\begin{aligned}\Delta Q &= mc\Delta T \\ &= 30 \times 3470 \times (37 - 39) \\ &= -2.1 \times 10^5 \text{ J}\end{aligned}$$

Heat: Measurement of specific heat by mixing method

Measurement of specific heat by mixing method:

When two substances with two different temperatures T_1 and T_2 are mixed, an equilibrium temperature T is then reach by both of them. We can therefore write the following relationship:

The amount of heat gained = the amount of heat lost

$$Q_{in} = Q_{out}$$

$$\because \Delta Q = mc\Delta T$$

$$m_1 c_1 (T - T_1) = m_2 c_2 (T_2 - T)$$

Where m_1, c_1, T_1: temperature , specific heat , and mass of the first substance.

and m_2, c_2, T_2: temperature , specific heat , and mass of the 2nd substance.

Heat: Example 6

Example 6:

A thermos kettle contains 300 g of water at 90^oC . Pouring into this kettle 50 g of water at 15^oC . What is the final temperature of the mixture?

Solution:

$m_1c_1(T - T_1) = m_2c_2(T_2 - T)$

$50\ (T - 15) = 300\ (90 - T)$

$T - 15 = 540 - 6T$

$7T$=555

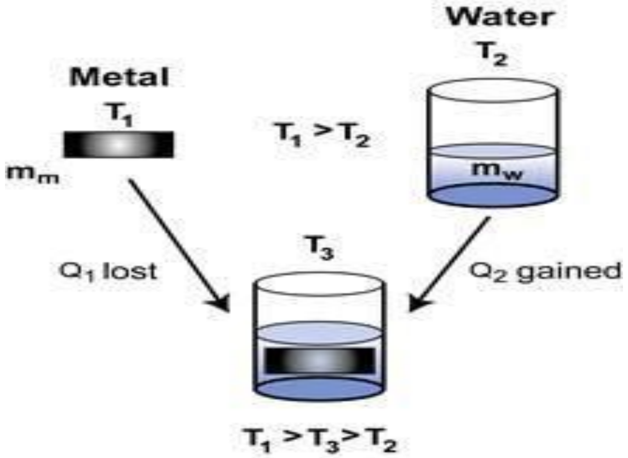
T = 79.3^oC

Heat: Example 7

Example 7:

An insulated aluminum pot ($c = 0.21\ \frac{cal}{g.^oC}$) container of mass 20 g contains 150 g of water at 20^oC. A 30-g piece of metal is heated to a point 100^oC and dropped into water. If the final temperature of the water, the vessel, and the piece of metal is 25^oC , what is the specific heat capacity of the metal?

Solution:



heat gained by pot + heat gained by water = Heat lost by metal

$$m_p c_p (T_3 - T_1) + m_w c_w (T_3 - T_1) = m_m c_m (T_2 - T_3)$$

$$20 \times 0.21(25 - 20) + 150 \times 1(25 - 20) = 30 \times c_m (100 - 25)$$

$$\Longrightarrow 21 + 750 = 2250 c_m$$

$$\Longrightarrow c_m = 0.343\ \frac{cal}{g.^oC}$$

Heat: Latent heat of fusion and vaporization

Latent heat of fusion and vaporization :

When the state of a substance changes from solid to liquid or from liquid to gaseous, this is done after providing the material with sufficient heat energy to change its state, which occurs with a constant temperature. To calculate the amount of heat (Q) required to perform the transformation of a mass of substance m, we use the following relationships:

First : Heat required for a substance to change from solid to liquid (or vice versa):

$$\Delta Q = mL_f$$

Where L_f is the latent heat of fusion and its unit is: $\frac{J}{kg}$ or $\frac{cal}{g}$

Second: Heat required for a substance to change from liquid to gas (or vice versa):

$$\Delta Q = mL_v$$

Where L_v is the latent heat of vaporization and its unit is: $\frac{J}{kg}$ or $\frac{cal}{g}$

Heat: Example 8

Example 8:

A container contains a mass of 0.25 kg of water at a temperature 20^oC . The container is placed in a freezer. Calculate the heat that must be removed from water in order for it to convert it into ice at a temperature 0^oC ,if you know that the latent heat of fusion of water is $334\,\frac{kJ}{kg}$ and the specific heat of water is: $4.2\,\frac{kJ}{kg}$

Solution:

$$\Delta Q = mc(T_f - T_i) + (-mL_f)$$

$$\Delta Q = m_w c_w (T_f - T_i) + (-m_w L_f)$$

$$= 0.25 \times 4.2 \times 10^3 \times (0 - 20) + (-0.25 \times 334 \times 10^3)$$

$$= -1.045 \times 10^5\ kJ$$

Heat: Example 9

Example 9:

Calculate the heat required to evaporate 10 kg of water if you know that the latent heat of evaporation of water is 2256 kJ/kg at atmospheric pressure and 100 °C.

Solution:

$$\Delta Q = mL_v$$

$$\Delta Q = 10 \times 2256 \times 10^3$$

$$\Delta Q = 2.256 \times 10^7 \text{ J}$$

Heat: Thermal expansion

Longitudinal thermal expansion:

Temperature is a measure of the internal energy of the molecules of a substance. When the temperature of a solid or liquid is raised, the energy of its molecules increases, which increases the amplitude of their vibrations. *This leads to an increase in the average distance between molecules, causing the substance to expand when heated*.

The coefficient of linear thermal expansion (α) is defined as the fractional increase in length per unit length of a material for each degree of temperature change. It is expressed as:

$\alpha = \Delta L / (L_0 \Delta T)$

where L_0 is the initial length, ΔL is the change in length, $\Delta L = L - L_0$,and ΔT is the temperature change $\Delta T = T - T_0$. The unit of α is $°C^{-1}$ or K^{-1}.

If α is constant over the temperature range, then the length at temperature T is:

$L(T) = L_0 (1 + \alpha \Delta T)$

Heat: Example 10

Example 10:

A copper rod has a length of 1 m. What is the increase in its length when its temperature rises by 50 °C above room temperature? The coefficient of linear expansion of copper is $\alpha = 1.9 \times 10^{-5}\ °C^{-1}$.

Solution:

$$\Delta L = \alpha\, L_0\, \Delta T$$

$$\Delta L = (1.9 \times 10^{-5})(1)(50)$$

$$\Delta L = 0.00095\ \text{m} = 0.95\ \text{mm}$$

Heat: Thermal expansion

Volumetric thermal expansion :

The volumetric expansion coefficient (γ) is the relative change in volume per unit change in temperature. It is defined by the relation:

$\gamma = \Delta V / (V_0 \, \Delta T)$

where V_0 is the initial (reference) volume at temperature T_0 , $\Delta V = V - V_0$ is the change in volume, and $\Delta T = T - T_0$ is the temperature change. The unit of γ is $°C^{-1}$ or K^{-1}.

If γ is constant over the temperature range, then:

$V(T) = V_0 \, (1 + \gamma \, \Delta T)$

Heat: Example 11

Example 11:

An aluminum sphere has a volume of 113 mm^3 at a temperature of 100 °C. What is its volume at 0 °C if the volumetric expansion coefficient of aluminum is $\gamma = 7.2 \times 10^{-5}$ $°C^{-1}$?

Solution:

$$V_{100} = V_0\,(1 + \gamma\,\Delta T)$$

$$V_0 = V_{100} \,/\, (1 + \gamma\,\Delta T)$$

$$V_0 = 113 \,/\, (1 + (7.2 \times 10^{-5})(100))$$

$$V_0 = 113 \,/\, 1.0072 \approx 112.2\ \mathrm{mm}^3$$

Heat: Thermal expansion

Anomalous Expansion of water :

Most substances contract when cooled, but **water behaves abnormally** between **0 °C and 4 °C**. When cooled from **4 °C down to 0 °C**, water **expands** instead of contracting.

At **4 °C**, water reaches its **maximum density**.

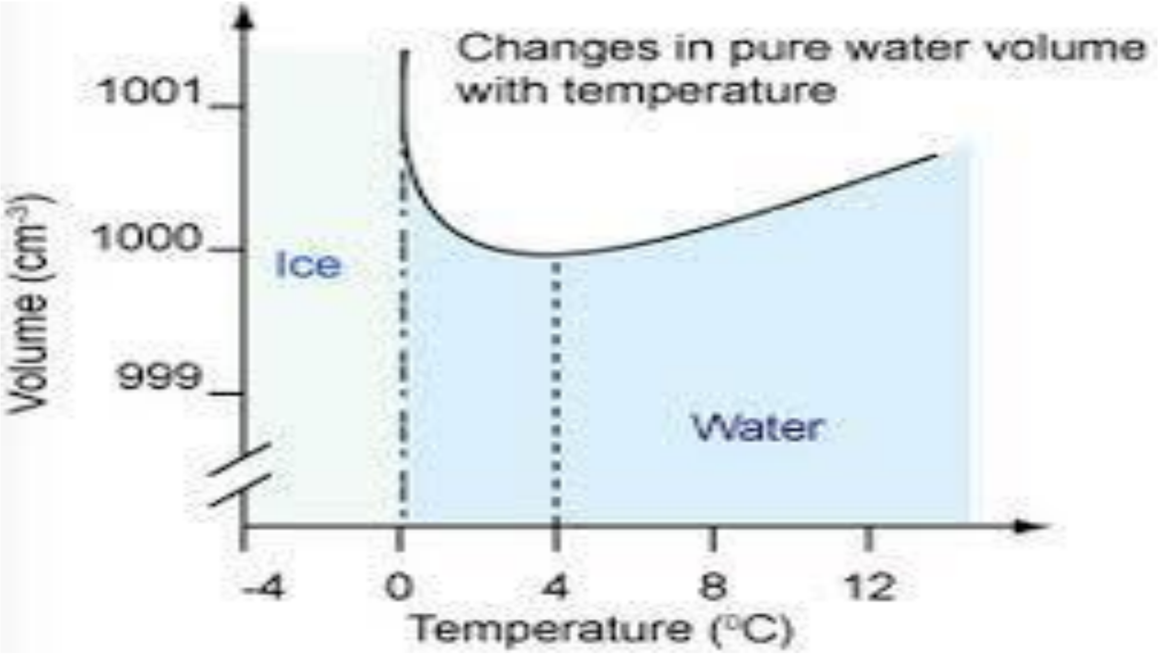
This has an important consequence in lakes and rivers:

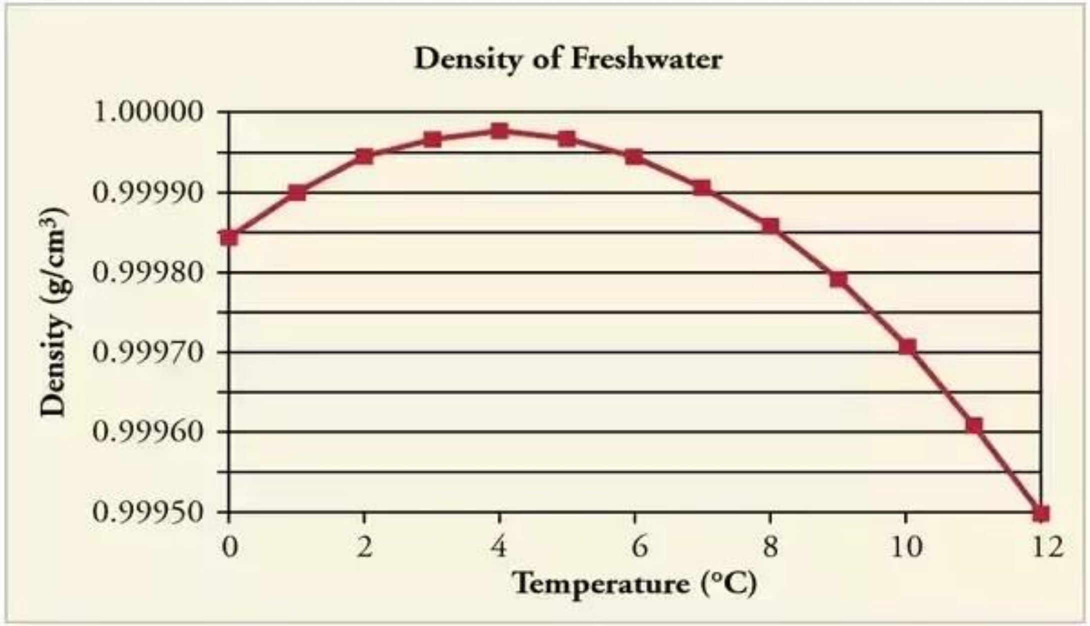
- Water at **4 °C (heaviest)** sinks to the bottom.
- Colder water (0–4 °C) and ice (less dense) stay near the surface.
- As a result, the surface freezes while deeper water remains liquid, allowing aquatic life (fish, and other organisms living in water) to survive in winter.

Ice also insulates the water below because its ***thermal conductivity is about 4 times lower*** *than that of liquid water.*

Heat: Thermal expansion

Anomalous Expansion of water :





Heat: Thermal expansion

Anomalous Expansion of water :

Why Pipes Crack :

- Most liquids contract when cooled, but **water is unusual**: it expands when cooled below **4 °C**.
- If the pipe is completely filled and closed, there's **no space to accommodate the expansion**.
- At **0 °C**, water freezes into ice and expands sharply by about **9%**, creating pressures of up to **~200 MPa (~2,000 atm)** when confined inside a sealed pipe.
- This produces **very high internal pressure** inside the pipe.
- Since copper pipes are rigid and cannot stretch much, they **crack or burst**.

Heat: Thermal expansion

Anomalous Expansion of water :

Why Pipes Crack :

Copper pipes are less common in new installations; flexible plastic pipes like PEX or PVC are widely used today because they better withstand freezing and are easier to install.

Thanks

King Saud University

College of Science

Phys 105

Physics for Architecture Students

Heat

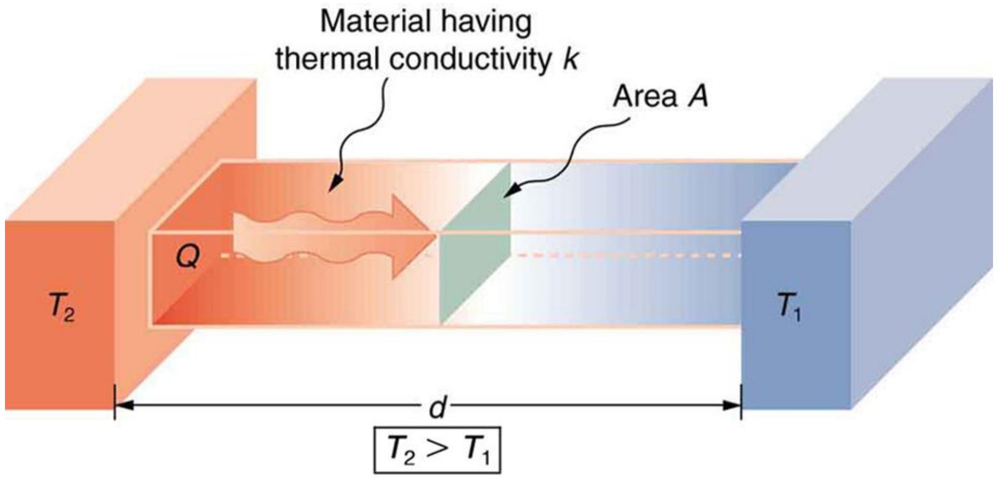
Chapter 19

Heat Transfer

Heat Transfer: Heat transfer methods

Heat transfer methods:

Heat travels from one place or body to another in three different ways: *conduction*, *convection* and *radiation*.



<u>**First**</u>: Heat transfer by *conduction*:

Thermal conduction occurs when there is a difference in temperature through the material. It was found practically that the rate of heat flow through the material is directly proportional to the difference in the two end temperatures. The rate of heat flow also depends on the size and shape of the material.

Heat Transfer: Heat transfer methods: Flow Rate

Heat transfer methods: Flow Rate

The heat flow rate ΔQ/Δt can be written by:

$$\frac{\Delta Q}{\Delta t} = kA\frac{T_1 - T_2}{L}$$

where A is the cross-sectional area of the body.

L is the distance between the ends of the body

T_1 and T_2 are temperatures of the two ends of the body ($T_1 > T_2$)

K is the thermal conductivity constant of a substance and its unit is: $\frac{J}{m.s\ C}$

Heat Transfer: Example 1

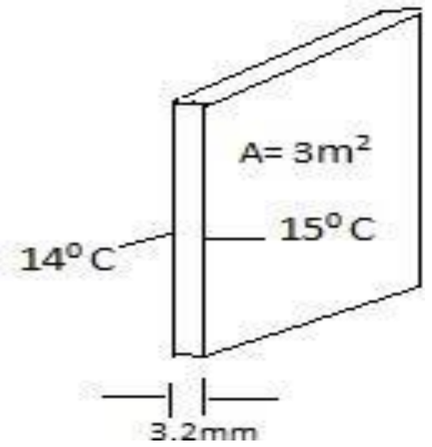
Example 1:

Calculate the rate of heat transfer through a window of a house of dimensions ($2m \times 1.5m$) and thickness (3.2 mm). If the temperatures on the inner and outer surfaces are 15^0 C and 14^0 C, respectively, and the thermal conductivity constant of the glass is $\frac{0.84 J}{m.s\ C}$

Solution:

$$A = 2 \times 1.5 = 3\text{m}^2$$

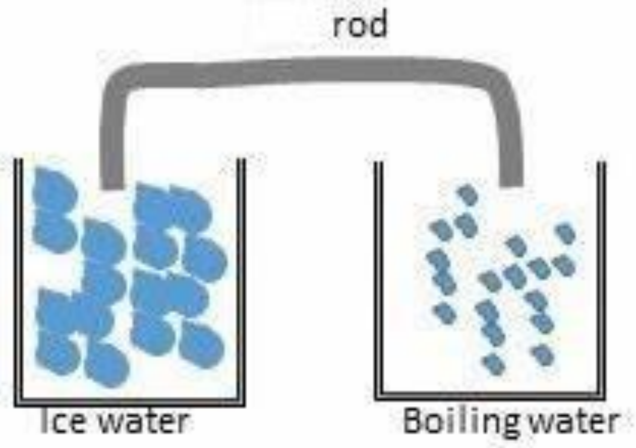
$$\frac{\Delta Q}{\Delta t} = kA\frac{T_1 - T_2}{L}$$
$$= \frac{0.84 \times 3 \times (15 - 14)}{3.2 \times 10^{-3}}$$
$$= 790\frac{J}{s} \cong 0.19\ kcal/s$$



Heat Transfer: Example 2

Example 2:

A brass rod has a cross-sectional area of 2 cm^2 and a length of 1 m. One end of this rod is placed in boiling water while the other one on a large piece of ice. How much ice is melted by the heat transferred from the hot end of the rod to the cold end during (10 min)? thermal conductivity constant of copper: $\frac{0.2cal}{cm.s\ C}$



Solution:

$$\frac{\Delta Q}{\Delta t} = kA\frac{T_1 - T_2}{L}$$

$$\rightarrow \Delta Q = kA\frac{T_1 - T_2}{L} \times \Delta t$$

$$= 0.2 \times 2 \times \frac{100-0}{100} \times (10 \times 60) = 240 \text{ cal}$$

Since the latent heat of fusion of snow L_f is equal to $\frac{80\ cal}{g}$,the amount of melted snow, m, is:

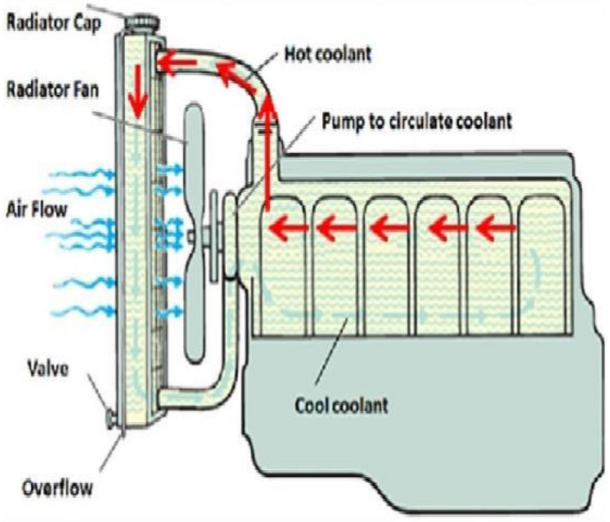
$$m = \frac{\Delta Q}{L_f} = \frac{240}{80} = 3\ g$$

Heat Transfer: Heat transfer methods

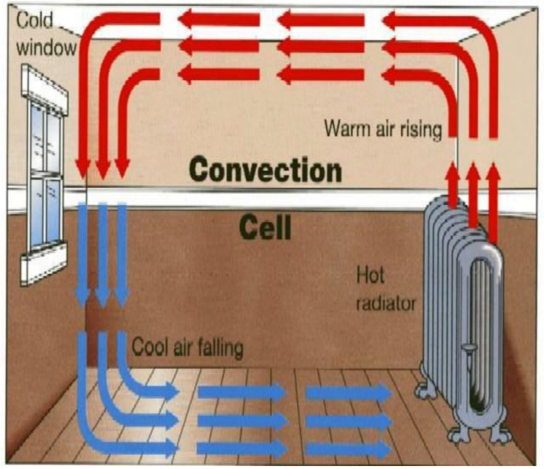
Heat transfer methods:

2nd : Heat transfer by *convection* :

- **Convection** is the transfer of heat in **liquids and gases** through the **bulk movement of particles**.
- Unlike conduction (where energy passes molecule to molecule), in convection the **fluid itself moves**, carrying heat from one place to another.
- This allows heat to spread over large distances in the medium.



Forced Convection



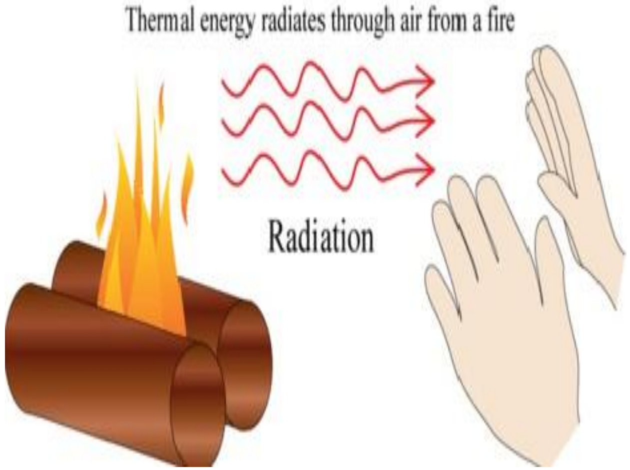
Natural Convection

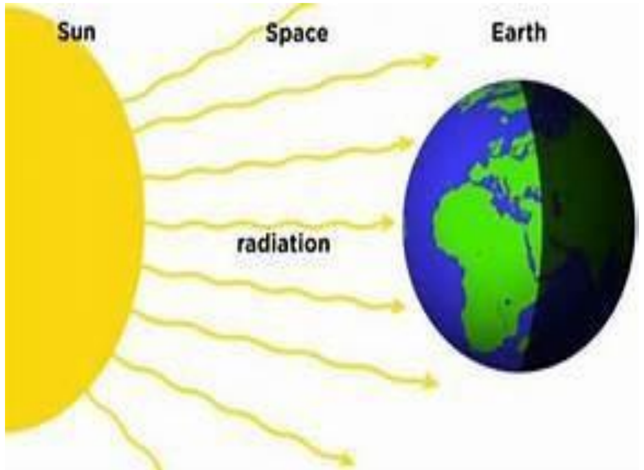
Heat Transfer: Heat transfer methods

Heat transfer methods:

3rd : Heat transfer by *radiation* :

- **Radiation** is the transfer of heat energy by **electromagnetic waves** (mainly infrared).
- Unlike conduction and convection, **radiation does not require matter** — it can occur through a **vacuum**.
- Example: The heat we receive from the **Sun** travels through empty space by radiation.





Heat Transfer: Stefan–Boltzmann Law

Stefan–Boltzmann Law :

The rate of *heat transfer* by radiation from a *hot body* is given by a relationship called Stefan–Boltzmann Law, which is:

$$\frac{\Delta Q}{\Delta t} = \sigma e A T^4$$

The unit of $\frac{\Delta Q}{\Delta t}$ is the power radiated (unit: watt, W = J/s)

A :is the surface area of the body

T :is the absolute temperature of the body (in kelvins)

e : **emissivity** (0 ≤ e ≤ 1), a measure of how efficiently a surface radiates heat (e=1 for a perfect blackbody)

σ : Stefan–Boltzmann constant = $5.67 \times 10^{-8} \frac{W}{m^2 K^4}$

Heat Transfer: Stefan–Boltzmann Law

Net Radiation:

If the body is surrounded by another body (or medium) at absolute temperature T_0, then:

$$\frac{\Delta Q}{\Delta t} = \sigma e A(T^4 - T_0^4)$$

This formula shows that heat transfer depends on the **difference in the fourth powers** of the temperatures.

Heat Transfer: Stefan–Boltzmann Law

Net Radiation:

It has been proven that a black body one that does not reflect light) radiates a greater amount of heat compared to other bodies with higher reflectivity. As a general rule it can be said that a good heat absorber is a good heat radiator.

- A blackbody (perfect absorber, e=1) gives off the maximum possible radiation at any given temperature.
- Good absorbers of heat are also good radiators . For example, a dark roof absorbs more sunlight during the day and radiates more heat at night.
- Shiny or reflective surfaces (like polished metal) are poor absorbers and therefore also poor radiators — this is why thermos flasks and reflective building materials help reduce heat gain or loss.

Thanks