Math 244 Linear Algebra
Semester 481, Exam 1
Time allowed: 90 minutes.

25 pts

KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

NAME: **Student ID:** **Section Number:**

Note: Calculators are not allowed.

Question	1	2	3	4	5	6	7	8	9	10	
Answer											10 pts

Question 1 Write the letter of the correct choice in the table provided.

(1) Any upper triangular matrix A satisfies

a) AA^T is upper triangular
b) $A - A^T$ is diagonal
c) A^2 is upper triangular
d) $A^2 - I$ is invertible

(2) Any real diagonal matrix A satisfies

a) A is invertible b) $A^2 + I$ is invertible c) $A - I$ is invertible d) $A + I$ is invertible

(3) The solution of the linear system $\begin{cases} -3x + y = -1 \\ 5x - 2y = 1 \end{cases}$ is:

a) $(1,2)$ b) $(-2,1)$ c) $(1,-1)$ d) $(3,3)$

(4) If A is a square matrix such that $A^2 + A = I$, then:

a) $A^{-1} = I - A$ b) $A^{-1} = A + I$ c) $A^2 = I$ d) A is singular

(5) The set of solutions of $Ax = 0$, where $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is:

a) $\{(t,0,0): t \in \mathbb{R}\}$ b) $\{(t,t,0): t \in \mathbb{R}\}$ c) $\{(0,0,t): t \in \mathbb{R}\}$ d) $\{(0,t,0): t \in \mathbb{R}\}$

(6) The matrix $\begin{bmatrix} 3 & 6 \\ 1 & 2x \end{bmatrix}$ is singular if:

a) $x = 3$ b) $x = 2$ c) $x = 1$ d) $x = 0$

(7) Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 4 & 0 \\ 1 & -1 & -2 \end{bmatrix}$ and $EA = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 4 & 0 \\ -1 & 2 & -3 \end{bmatrix}$. Then the elementary matrix E is equal to:

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(8) Let A be an invertible matrix. Which of the following matrices must be **both** invertible and symmetric?

a) $A - A^T$ b) AA^T c) $A + A^T$ d) A^{-1}

(9) Let A be a 3×3 matrix. If the system $Ax = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ has no solution, then:

a) $Ax = 0$ has infinitely many solutions
b) $Ax = 0$ has a unique solution
c) $A^T x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ has a unique solution
d) $Ax = 0$ has no solution

(10) Let A and B be 3×3 matrices with $|A| = 2$ and $|B| = -3$. Then $|2B^T A^2 B^{-1}|$ is equal to:

a) -32 b) 8 c) 16 d) 32

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Question 2 Consider the following system of linear equations: $\begin{cases} x_1 - 2x_2 + 3x_3 = 1 \\ 2x_1 - 3x_2 + (m+5)x_3 = -1 \\ x_1 + (m-2)x_2 + 3x_3 = -2 \end{cases}$

5 pts

Find the values of m such that the system has:

(i) a unique solution; (ii) no solution; (iii) infinitely many solutions.

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Question 3 Solve the following homogeneous linear system: $\begin{cases} x_1 - x_2 + 2x_3 + x_4 = 0 \\ -x_1 + x_3 - 3x_4 = 0 \\ x_1 - 2x_2 + 5x_3 - x_4 = 0 \\ -x_2 + 3x_3 - 2x_4 = 0 \end{cases}$

3 pts

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Question 4 If $A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 2 \end{bmatrix}$, find: (i) $(A^T)^{-1}$; (ii) adj(A).

3 pts

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Question 5 Consider the following linear system: $\begin{cases} \alpha x - y - z = -1 \\ -x + \alpha y + z = 1 \\ x + 2y + z = 1 \end{cases}$

4 pts

(i) Find the values of α for which the above system can be solved using Cramer's rule.

(ii) Assuming the system can be solved by Cramer's rule, find the value of y.

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