Note:

- **Organize your answers in the same order as the problems.**
- **Write legibly.**

1- Prove that $\left|z_1 + z_2\right| \leq \left|z_1\right| + \left|z_2\right|$, then generalize to $\left|z_1 + z_2 + \ldots + z_n\right| \leq \left|z_1\right| + \left|z_2\right| + \ldots + \left|z_n\right|$, $n \geq 2$.

2- If $f(z)$ and $g(z)$ are continuous at z_0 , then show that $f(z).g(z)$ is continuous at z_0 .

3- Show that $f(z) = \left|z\right|^2$ is continuous for all $z \in \mathbb{C}$, but not differentiable for all $z \neq 0$.

4- Find the solutions of $z^3 = -8i$, then draw them in the complex plane.

5- Show that $u(x,y) = x^4 - 6x^2y^2 + y^4$ is harmonic in $\mathbb{R}^2$, then find its harmonic conjugate $v(x,y)$. Write the resulting analytic function $f(z) = u(x,y) + iv(x,y)$ in terms of z .