Differential and Integral Calculus (MATH-205)

Final Exam/Sem I (2022-23) Time Allowed: 180 Minutes

Date: Tuesday, November 15, 2022 **Maximum Marks:** 40

Note: Attempt all **9** questions and give **DETAILED** solutions. Make sure your solutions are clearly written and contain all necessary details.

Question 1: (4°) Determine whether the following infinite series converges or diverges.

$$\sum_{n=1}^{\infty}\left(1+\frac{2}{n}\right)^{n^2}$$

Question 2: (5°) Find a power series representation of $f(x) = \tan^{-1} x$. Specify the radius and interval of convergence of the series. Hence, prove that

$$\frac{\pi}{6} = \sum_{n=0}^{\infty}(-1)^n \frac{1}{3^{n+\frac{1}{2}}(2n+1)}$$

Question 3: (4°) Show that p_1 and p_2 are two parallel planes. Find the distance between the them.

$$p_1 : \; 3x + 12y - 6z = -2, \qquad p_2 : \; 5x + 20y - 10z = 7$$

Question 4: (3°) If $\mathbf{a}$ and $\mathbf{b}$ are any nonzero vectors in $\mathbb{R}^3$. Under what condition, we have $\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 = \|\mathbf{a}+\mathbf{b}\|^2$? Explain with reasoning.

Question 5: (4°) Identify and describe the surface: $x^2 + 16\,y + 4\,z^2 = 0$. Find and describe its traces in xy-, yz-, and xz-planes.

Question 6: (5°) Let $z = f(x, y)$ be defined implicitly as a function of x and y by the equation

$$x^2 + 2y + 3z^2 = 2.$$

Find the directional derivative of f at $(0, 0)$ in the direction of maximum increase in f.

—- PTO —-

Question 7: (6°) Find the local and boundary extrema and saddle points of $f(x,y) = \frac{1}{2}x^2 + 2xy - \frac{1}{2}y^2 - 8y + x$ on R bounded by the graphs of $y = -x$, $y = x$, and $x = 4$. Sketch R.

Question 8: (4°) Evaluate the double integral $\iint\limits_R y\,x^2\,dA$, where R is the region bounded between the graphs of $y = \sqrt{x}$ and $y = 1 - x$ from $x = 1$ to $x = 2$. Sketch the region R.

Question 9: (5°) Find the volume V of the solid that lies under the graph of the equation $z = x^2 + 4y^2$ and over the region on the xy-plane bounded by the polygon with the vertices at $(0,0)$, $(2,1)$, and $(-2,1)$. Sketch the polygon region.

—- Good Luck —-