Differential and Integral Calculus (MATH-205)

Final Exam/Sem II (2022-23) Time Allowed: 3 Hours

Date: Tuesday, February 21, 2023 **Maximum Marks:** 40

Note: Solve all **9** questions and give **DETAILED** solutions. Make sure your solutions are clearly written and contain all necessary details.

Question 1: (5°) Show that the following infinite series converges and find its sum.

$$\sum_{n=0}^{\infty}\left(\frac{9}{(3n+1)(3n+4)}-\frac{2^n}{3^{n+1}}\right)$$

Question 2: (5°) Find a power series representation of $f(x)=\ln(8+x^3)$. Specify the radius and interval of convergence of the series.

Question 3: (3°) A constant force of magnitude 4 lb has the same direction as the vector $\mathbf{a}=\hat{i}+\hat{j}+\hat{k}$. If distance is measured in feet, find the work done if the point of application moves along the $y-$axis from $(0,2,0)$ to $(0,-1,0)$. Find the component of the force along the direction of displacement and interpret why the work done is negative.

Question 4: (4°) Let the lines l_1 and l_2 have the respective parametrizations given by

$$l_1: \quad x=4+5t, \quad y=3+2t, \quad z=3t, \quad t\in\mathbb{R}$$

$$l_2: \quad x=-5+2v, \quad y=4-v, \quad z=1, \quad v\in\mathbb{R}$$

Determine whether l_1 and l_2 are parallel, intersecting, or skew lines.

Question 5: (5°) Find the extrema of $f(x,y,z)=2x^2+y^2+3z^2$ subject to the constraint $2x-3y-4z=49$. Is it a minimum or maximum?

Question 6: (3°) Let $w=2xy$, where $x=s^2+t^2$ and $y=\frac{s}{t}$. Find w_s, w_t and w_{st}. Give your answers in terms of s and t in simplified form.

—- PTO —-

Question 7: (5°) Let $z = f(x, y)$ be defined implicitly as a function of x and y by the equation

$$x^2 - 2y^2 - z^2 = 0.$$

Find the directional derivative of f at $(-\frac{3}{4}, 0)$ in the direction of maximum increase in f.

Question 8: (5°) Evaluate the double integral $\iint\limits_R (x^2 + y^2)\, dA$, where R is the region bounded between the graphs of $x - y + 1 = 0$ and $x + y + 1 = 0$ and $x = 0$. Sketch the region R.

Question 9: (5°) Find the volume V of the solid that lies under the graph of the equation $z = x^2 + 4$ and over the region R in the xy-plane bounded by the graphs of $x = 4 - y^2$ and $x + y = 2$. Sketch the region.

—- Good Luck —-