Self

King Saud University,
Department of Mathematics.

M203 (Differential and integral Calculus),
Final Examination/ Second Sem. 1446

Max. Marks: 40 Marks: [Q1) 4+4+4, Q2) 4+4+4, Q3) 4+4+4+4] Time: 3 hs.

Q 1. (a) Find the sum of the series $\sum_{n=0}^{\infty} \frac{5 \cdot 2^{3n+1} - 2 \cdot 3^{2n+1}}{12^n}$, if it exists.

(b) Find the interval of convergence and the radius of convergence of the power series: $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n \cdot 3^n}$.

(c) Find the Maclaurin series of the function $f(x) = e^x$ and use it to obtain a Maclaurin series of the function xe^{3-2x}.

Q 2. (a) By reversing the order, evaluate the double integral

$$\int_0^2 \int_{y^2}^4 y \cos(x^2) dx dy.$$

(b) Find the mass of the lamina that has the shape of the region bounded by the graphs of the equations $y = 4 - x^2$, $y = 0$, and density at any point (x, y) is directly proportional to the distance between (x, y) and the x-axis.

(c) Evaluate the triple integral by changing it to spherical coordinates:

$$\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} y\sqrt{x^2 + y^2 + z^2} dz dy dx.$$

Q 3. (a) Find the work done by the vector force $\vec{F}(x, y) = y\vec{i} + x\vec{j}$ in moving a particle from $(1, -1)$ to $(1, 1)$ along the curve $x = y^2$.

(b) Show that the following line integral is independent of path and find its value: $\int_{(0,1)}^{(1,0)} [2y^3 \cos(x) + e^x - 3]dx + [6y^2 \sin(x) + 3e^y - 5]dy$.

(c) Find the flux of the force $\vec{F}(x, y, z) = 2\vec{i} + 3\vec{j} + z\vec{k}$ through the surface S of the solid bounded by the graphs of $z = x^2+y^2$ and $z = 2-x^2-y^2$.

(d) Verify Stokes theorem for the force $\vec{F}(x, y, z) = x\vec{i} - y\vec{j} + z\vec{k}$ and the surface S that is the portion of the paraboloid $z = x^2+y^2$ with boundary curve C having parametric equations $x = \cos(t), y = 1 + \sin(t)$, $z = 2 + 2\sin(t)$, $0 \leq t \leq 2\pi$.