Differential and Integral Calculus (MATH-205)

Final Exam/Fall 2021   Time Allowed: 180 Minutes

Date: Tuesday, December 28, 2021   **Maximum Marks:** 40

Note: Attempt all 9 questions and give detailed solutions. Make sure your solutions are clearly written and contain all necessary details.

Question 1: ($5°$) Find all values of x for which the following series converges, and find the sum of the series.

$$3 + (x-1) + \frac{(x-1)^2}{3} + \cdots + \frac{(x-1)^n}{3^{n-1}} + \cdots$$

Question 2: ($5°$) Find the interval and radius of convergence of the following power series

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^3} x^n.$$

Question 3: ($5°$) Given $f(x) = \frac{1}{3-2x}$, $|x| < \frac{3}{2}$. Find power series representation of $f(x)$, $f'(x)$, and $\int_0^x f(t)\, dt$.

Question 4: ($3°$) Find all values of α such that $\mathbf{a} = \langle 4, 2, \alpha \rangle$ and $\mathbf{b} = \langle 1, 22, -3\alpha \rangle$ are orthogonal vectors.

Question 5: ($4°$) Find an equation of the plane passing through the origin and the points $P(0, 2, 5)$ and $Q(1, 4, 0)$. Find the distance of the point $P(1, -1, 3)$ from this plane.

Question 6: ($3°$) Using two-path rule, show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{3x\,y}{5x^4 + 2y^4}$$

Question 7: (5°) Use Lagrange multipliers method to find the extrema of $f(x, y) = 2x^2 + xy - y^2 + y$ subject to $2x + 3y = 1$.

Question 8: (5°) Evaluate the double integral $\int_1^2 \int_{x^3}^x e^{\frac{y}{x}} \, dy \, dx$.

Question 9: (5°) Sketch the region R and evaluate the following double integral

$$\iint\limits_R \frac{x^2}{x^2 + y^2} \, dA,$$

where R is the region bounded by $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ with $0 < a < b$.

—- Good Luck —-