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Final Exam	Math 106	December 2025

Question 1 : (2+3+3)

a) If $F(x) = \int_2^{\ln x} (1+e^t)^4 dt$, find $F'(3)$.

b) Evaluate $\int \frac{3^x}{25+3^{2x}} dx$.

c) Compute $\int \frac{dx}{\sqrt{1-e^{6x}}}$.

Question 2 : (3+3+3)

a) Find $\lim_{x\to+\infty} (1+4x^2)^{\frac{1}{x^2}}$.

b) Compute $\int x^2 \cos(4x) dx$.

c) Evaluate $\int \tan^2 x \sec^6 x dx$.

Question 3 : (3+3+3)

a) Find $\int \frac{x^3+2x-1}{x^2-1} dx$.

b) Evaluate $\int \frac{1+x^2}{x^2\sqrt{x^2-9}} dx$.

c) Compute $\int \frac{\sec^2 x}{3+2\tan x+\tan^2 x} dx$.

Question 4 : (3+3+3)

a) Does the integral $\displaystyle\int_0^2 \frac{x}{(4-x^2)^{\frac{5}{2}}}dx$ converge?

Find its value if it does.

b) Sketch the region bounded by the curves $y = 2 - x^2$, $y = 2x - 1$, and find its area.

c) Find the volume of the solid obtained by revolving the region bounded by $y = \ln x$, $y = 0$, $x = 1$, $x = e$ about the $y-$axis.

Question 5 : (2+3)

1. Find the arc length of the curve with parametric equations $x = t - \sqrt{2}$, $y = \cosh t$, $0 \le t \le \ln 2$.

2. Sketch the region inside the graph of $r = 2 + 2\cos\theta$ and outside $r = 2$ and find its area.

Question 1 :

a) $F'(x) = \frac{1}{x}(1+x)^4$, **(1.5)**

$F'(3) = \frac{4^4}{3}$. **(0.5)**

b)

$$\int \frac{3^x}{25+3^{2x}}dx \overset{u=3^x}{=} \frac{1}{\ln 3}\int \frac{du}{25+u^2} \quad \textbf{(2)}$$
$$= \frac{1}{5\ln 3}\tan^{-1}(\frac{3^x}{5}) + c. \quad \textbf{(1)}$$

c)

$$\int \frac{dx}{\sqrt{1-e^{6x}}} \overset{u=e^{3x}}{=} \frac{1}{3}\int \frac{du}{u\sqrt{1-u^2}} \quad \textbf{(2)}$$
$$= -\frac{1}{3}\text{sech}^{-1}(e^{3x}) + c. \quad \textbf{(1)}$$

Question 2 : (3+3+3)

a)

$$\lim_{x\to+\infty}(1+4x^2)^{\frac{1}{x^2}} = \lim_{x\to+\infty} e^{\frac{\ln(1+4x^2)}{x^2}} \quad \textbf{(1)}$$
$$= \lim_{x\to+\infty} e^{\frac{4}{1+4x^2}} = 1. \quad \textbf{(2)}$$

b)

$$\int x^2\cos(4x)dx = \frac{x^2}{4}\sin(4x) - \frac{1}{2}\int x\sin(4x)dx \quad \textbf{(1.5)}$$
$$= \frac{x^2}{4}\sin(4x) + \frac{x}{8}\cos(4x) - \frac{1}{32}\sin(4x) + c. \quad \textbf{(1.5)}$$

1.

$$\int \tan^2 x\sec^6 x dx \overset{u=\tan x}{=} \int u^2(u^2+1)^2 du \quad \textbf{(2)}$$
$$= \frac{\tan^7 x}{7} + \frac{2}{5}\tan^5 x + \frac{1}{3}\tan^3 x + c. \quad \textbf{(1)}$$

Question 3 : (3+3+3)

a)

$$\int \frac{x^3+2x-1}{x^2-1}dx = \int \left(x+\frac{2}{x+1}+\frac{1}{x-1}\right)dx \quad \textbf{(1.5)}$$
$$= \frac{x^2}{2}+2\ln|x+1|+\ln|x-1|+c. \quad \textbf{(1.5)}$$

b)

$$\int \frac{1+x^2}{x^2\sqrt{x^2-9}}dx \overset{x=3\sec\theta}{=} \frac{1}{9}\int \cos\theta d\theta + \int \sec\theta d\theta \quad \textbf{(1)}$$
$$= \frac{1}{9}\sin\theta + \ln(\sec\theta+\tan\theta)+c \quad \textbf{(1)}$$
$$= \frac{\sqrt{x^2-9}}{9x}+\ln\left|x+\sqrt{x^2-9}\right|+c. \quad \textbf{(1)}$$

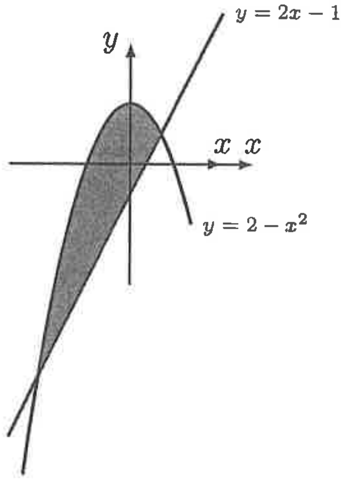
c)

$$\int \frac{\sec^2 x}{3+2\tan x+\tan^2 x}dx \overset{u=\tan x}{=} \int \frac{du}{(u+1)^2+2} \quad \textbf{(2)}$$
$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u+1}{\sqrt{2}}\right)+c$$
$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x+1}{\sqrt{2}}\right)+c. \quad \textbf{(1)}$$

Question 4 : (3+3+3)

a) $\displaystyle\int \frac{x}{(4-x^2)^{\frac{5}{2}}}dx = -\frac{1}{2}\int \frac{-2x}{(4-x^2)^{\frac{5}{2}}}dx = \frac{1}{3(4-x^2)^{\frac{3}{2}}}.$ **(2)**

Then the integral $\displaystyle\int_0^2 \frac{x}{(4-x^2)^{\frac{5}{2}}}dx$ is divergent. **(1)**



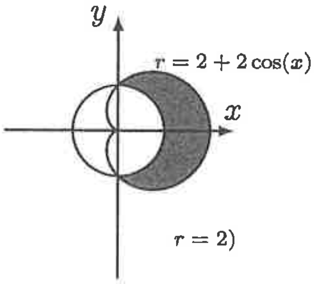
b) **(05)**

$$A = \int_{-3}^{1} (3 - x^2 - 2x)dx = 11 - \frac{1}{3}. \quad \textbf{(1.5)}$$

c) $V = 2\pi \int_1^e x \ln x dx \overset{\textbf{(2)}}{=} 2\pi[\frac{x^2 \ln x}{2} - \frac{x^2}{4}]_1^e = \frac{\pi}{2}(1 + e^2)$. **(1)**

Question 5 : (2+3)

1. $L \overset{(1)}{=} \displaystyle\int_0^{\ln 2} \sqrt{1 + \sinh^2 x} dx \overset{(1)}{=} \frac{3}{4}.$

2. 

Graph: (1)

$$\begin{aligned} A &= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left((2 + 2\cos\theta)^2 - 4\right) d\theta \quad (1) \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(4\cos\theta + 2\cos^2\theta\right) d\theta \\ &= 4\sin\theta + \theta + \frac{\sin(2\theta)}{2}]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \pi + 8 \quad (1) \end{aligned}$$