FINAL EXAMINATION, SEMESTER II, 1445

Department of Mathematics, College of Science

King Saud University

MATH 107 FULL MARK 40 TIME 3 HOURS

Q1. [Marks: $3+4=7$]

(a) Find the value of λ for which the following system of linear equations

$$3x+y+(\lambda^2-6)z=\lambda-3$$
$$2x+3y+z=-1$$
$$x+2y+z=0$$

has: (i) no solution (ii) unique solution (iii) infinitely many solutions.

(b) Use adjoint method to find the **inverse** of the coefficient matrix of the following system, and then using this inverse to find the solution of the system

$$x+2y+3z=1$$
$$2x-y=0$$
$$3x+3z=2$$

Q2: [Marks: $2+3+3+3=11$]

(a) If l has parametric equations $x=5-3t, y=-2+t, z=1+9t,$ find parametric equations for the line through $P(-6,4,-3)$ that is parallel to l.

(b) Find an equation of the plane through the point $P(2,4,-5)$ with normal $\mathbf{a}=-\mathbf{i}+2\mathbf{j}-3\mathbf{k}$.

(c) Find the volume of a box having adjacent sides $AB, AC,$ and AD where $A(2,1,-1), B(3,0,2), C(4,-2,1)$ and $D(5,-3,0)$.

(d) Sketch the graph of $4x^2-9y^2+z^2=36$ in an xyz - coordinate system, give traces and identify the surface.

Q3: [Marks: $3+4+3=10$]

(a) A moving particle starts at an initial position $\mathbf{r}(0)=\langle 1,0,0\rangle$ with initial vector $\mathbf{v}(0)=\mathbf{i}-\mathbf{j}+\mathbf{k}$. If its acceleration $\mathbf{a}(t)=4t\mathbf{i}+6\text{t}\,\mathbf{j}+\mathbf{k}$, find the velocity and position at time t.

(b) Find tangential component of acceleration and normal component of acceleration for the curve given by $\mathbf{r}(t)=2t\mathbf{i}+t^2\mathbf{j}-\frac{1}{3}t^3\mathbf{k}$. Also, find curvature.

(c) Let $f(x,y)=\dfrac{xy^2}{x^2+y^4}$. Show that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

Q4: [Marks: $3+3+3+3=12$]

(a) Find the equation of the tangent plane, and the parametric equations for the normal line to the graph of $x^3-2xy+z^3+7y+6=0$ at the point $P(1,4,-3)$.

(b) Find the directional derivative of $f(x,y,z)=xy+yz+xz$ at the point $A(1,1,1)$ in the direction of the vector $\mathbf{v}=2\mathbf{i}+\mathbf{j}-\mathbf{k}$.

(c) If $g(x,y)=-x^3+4xy-2y^2+1$, find the local extrema and saddle points of g.

(d) Let $f(x,y,z)=2x^2+y^2+3z^2$. Use Lagrange multiplier to find minimum value of f subject to the constraint $2x-3y-4z=49$.