King Saud University | College of Sciences | Department of Mathematics

Final Examination | Math 228 | Second Semester (1446) | Time: 3 Hours

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Question I: (18 marks)

1. Evaluate $\iint\limits_D ye^{x^2} dA$, where D is the region bounded by the graphs $y = \sqrt{x}$, $y = 0$ and $x = 1$. **(3 marks)**
2. Evaluate the integral $\int\limits_0^1 \int\limits_{x^2}^1 x \cos y^2 \; dydx$.(Hint: reverse the order of the integration) **(3 marks)**
3. Evaluate the integral $\int\limits_{-1}^1 \int\limits_0^{\sqrt{1-x^2}} \sqrt{x^2+y^2}\, dydx$.(Hint: Use polar coordinates).**(3 marks)**
4. Evaluate the integral $\iiint\limits_Q (xy^2 + z) \; dV$,

   where $Q = \{(x, y, z) : 0 \leq x \leq 2, \; -1 \leq y \leq 1, \; 0 \leq z \leq 3\}$. **(3 marks)**
5. Use cylindrical coordinates to evaluate the integral $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^2 \sqrt{x^2+y^2}\, dz\, dy\, dx$. **(3 marks)**
6. Evaluate the integral $\iiint_E z \; dV$ by changing to spherical coordinates, where E is the upperhalf of the sphere $x^2 + y^2 + z^2 = 1$. **(3 marks)**

Question II: (15 marks)

1. Find the limit of the sequence $\left\{\frac{\ln n}{n}\right\}$. **(2 marks)**
2. Determine whether the series $\sum\limits_{n=1}^{\infty} \frac{2n^3 - n^2 + 1}{n^3 + n}$ converges or diverges. **(2 marks)**
3. Determine whether the series $\sum\limits_{n=1}^{\infty} \frac{2}{5^n}$ converges or diverges, and if it converges find its sum. **(3 marks)**
4. Determine whether the series $\sum\limits_{n=1}^{\infty} n^2 e^{-n^3}$ converges or diverges. (Hint: use the integral test) **(2 marks)**

5. Use the Limit Comparision Test to determine whether the series $\sum_{n=1}^{\infty} \frac{2n+3}{n^3+1}$ converges or diverges. **(3 marks)**

6. Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \frac{2^{n-1}}{3^n \sqrt{n+1}}$ is absolutely convergent, conditionally convergent, or divergent. **(3 marks)**

Question III: (7 marks)

1. Find the interval of convergence and the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n5^n}$. **(4 marks)**

2. Find the power series representation of the function $\frac{1}{1+x}$ where $|x| < 1$, and then use this result to find the power series representation of the function $\ln(1+x)$.**(3 marks)**