King Saud University
Department of Mathematics

Final Exam in Math 151, T2-1444H.
Calculators are definitely not allowed
(The exam is 2-pages long)

Q1. (a) Without using truth tables show that $[(p \to q) \wedge p] \to q$ is a tautology. (3)
(b) Use induction to show that $4 + 12 + 20 + \ldots + (8n - 4) = (2n)^2$ for all $n \geq 1$. (4)
(c) Suppose m and n are integers. Use a proof by contraposition to show that:
if $4 \mid (m^2 + n^2)$, then m or n is even. (3)

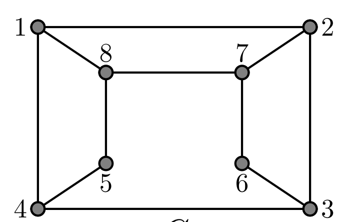
Q2. (a) Let E be the relation on $\mathbb{R} - \{0\}$ defined as follows:

$$xEy \text{ if and only if } \frac{x}{y} \text{ is a } \textbf{positive} \text{ rational number.}$$

(i) Show that E is an equivalence relation. (3)
(ii) Find $[1]$. (1)
(iii) Is $\sqrt{27}$ in $[-\sqrt{12}]$? (Justify your answer.) (1)

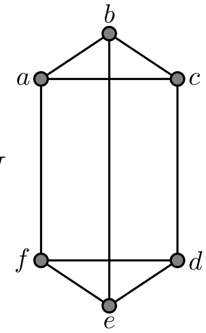
(b) Let $P = \{(1,1), (2,1), (2,2), (3,1), (3,3), (4,1), (4,4), (5,1), (5,5)\}$ be a relation on $A = \{1, 2, 3, 4, 5\}$.
(i) Show that P is a partial order. (3)
(ii) Is P a total order? (Justify your answer.) (1)
(iii) Represent P by a Hasse diagram. (1)

Q3. (a) Find the number of vertices of the complete bipartite graph $K_{n,2n^2}$ which has 16 edges. (2)
(b) Determine whether the following graph G is bipartite. If so, find a bipartite representation. (2)

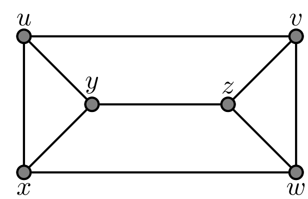


G

(c) Determine if the graphs H and I below are isomorphic. (2)



H



I

Q4. **(a)** For the graph I in **Q3(c)**, find a spanning tree with root v,

(i) using *depth-first* search; (1)

(ii) using *breadth-first* search. (1)

(b) Show that a tree with 12 vertices cannot be a complete graph. (1)

(c) Using alphabetical order, form a binary search tree for the words: *January, February, March, April, May, June, July, August.* (2)

Q5. **(a)** Without using tables, prove the following Boolean identity. (2)

$$\overline{x\overline{y} + y} = \overline{x + y}\,.$$

(b) Let $f(x, y, z) = (x + yz)(y + \overline{z})$ be a Boolean function.

(i) Find the complete sum-of-products expansion (CSP) of f. (2)

(ii) Find the complete product-of-sums expansion (CPS) of f. (2)

(c) Let $g(x, y, z) = xy\overline{z} + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}y\overline{z} + \overline{x}\,\overline{y}z$ be a Boolean function.

(i) Build the Karnaugh map of g. (1)

(ii) Simplify g (i.e., write it in MSP form). (2)