King Saud University
Department of Mathematics

Final Exam in Math151, Semester 2, 1443H.
Calculators are not allowed
(The exam is two-pages long)

Q1. (a) Without using truth tables show that $(p \to q) \to r$ is logically equivalent to $(\neg r \to p) \wedge (q \to r)$. (2pts)

(b) Use induction to show the following for every $n \geq 2$:

$$(\frac{1}{2} - \frac{1}{4})(\frac{1}{2} - \frac{1}{6})(\frac{1}{2} - \frac{1}{8}) \cdots (\frac{1}{2} - \frac{1}{2n}) = \frac{1}{2^{n-1}n}. \text{ (4pts)}$$

(c) Assuming that $\sqrt{6}$ is irrational, use a proof by way of contradiction to show that $\frac{5\sqrt{6}-3}{2} + 4$ is irrational. (2pts)

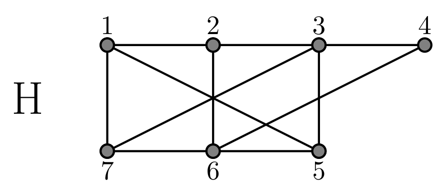
Q2. (a) Let R be the relation on $\mathbb{Z}$ (the set of integers) defined by mRn if and only if $2 \mid (m+n)$.
(i) Show that R is an equivalence relation. (3pts)
(ii) Find the equivalence classes [0] and [1]. (2pts)

(b) Let P be the relation on $\{1, 3, 5\}$ defined by: $aPb \iff a < b + 2$.
(i) List all ordered pairs of P. (2pts)
(ii) Represent P by a digraph. (1pts)
(iii) Show that P is a partial order. (3pts)
(iv) Is P a total order? (Justify your answer.) (1pts)
(v) Represent P by a Hasse diagram. (1pts)

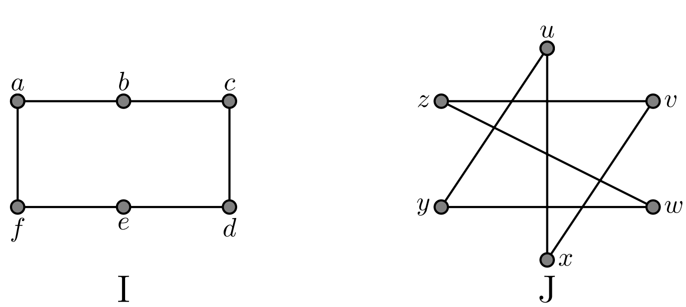
Q3. (a) Let G be a graph with degree-sequence: $a-3, a-2, a-1, a, a+2$. Find the value of a if G has 8 edges. (2pts)

(b) Let T be a tree with 12 edges. Find the number of vertices of T and the number of edges of the complement $\overline{T}$ of T. (2pts)

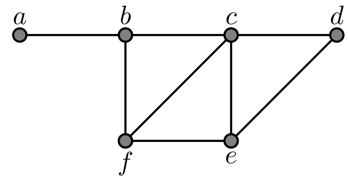
(c) Determine whether the graph H below is bipartite. If so, give a bipartite representation. (2pts)



(d) Determine whether the following graphs I and J are isomorphic. (2pts)



Q4. (a) For the graph L below, find a spanning tree with root b,



L

(i) using *depth-first* search; (1pts)
(ii) using *breadth-first* search. (1pts)

(b) Using alphabetical order, form a binary search tree for the words *purple, black, yellow, green, blue, white, grey.* (2 pts)

Q5. (a) Let $f(x, y, z) = \overline{\overline{x}y + \overline{y}z + \overline{y}}$ be a Boolean function.
(i) Find the complete sum-of-products expansion (CSP) of f. (2pts)
(ii) Find the complete product-of-sums expansion (CPS) of f. (2pts)

(b) Let $g(x, y, z, w) = xyzw + xyz\overline{w} + xy\overline{z}w + x\overline{y}\,\overline{z}w + \overline{x}\,\overline{y}zw + \overline{x}\,\overline{y}z\overline{w} + \overline{x}yzw + \overline{x}y\overline{z}\,\overline{w} + \overline{x}y\overline{z}w$ be a Boolean function.
(i) Build the Karnaugh map of g. (1pts)
(ii) Simplify g (i.e., write in MSP form). (2pts)