King Saud University
Department of Mathematics

Final Exam in Math 151
Semester 1, 1443 H.

Calculators are not allowed
The Examination contains 2 pages

Q1. (a) Without using truth tables show that $(p \wedge q) \vee (\neg p \wedge q) \vee (p \wedge \neg q) \equiv p \vee q$. (2pts)

(b) Suppose a, b and c are integers. Prove that if a divides $(b-1)$ and a divides $(c-1)$; then a divides $(bc-1)$. (3pts)

(c) Use induction to show that $1+5+9+\ldots+(4n-3) = n(2n-1)$ for all $n \geq 1$. (4pts)

Q2. (a) Let R be the relation on $\mathbb{Q}$ (the set of rational numbers) defined by xRy if and only if $x-y$ is an integer.

(i) Show that R is an equivalence relation. (3pts)

(ii) Find the equivalence class $[0]$. (1pts)

(iii) Show that $[7/16] \neq [-1/8]$. (1pts)

(b) Let $P = \{(a,a),(a,c),(b,b),(b,d),(c,c),(d,d)\}$ be a relation on the set $A = \{a,b,c,d\}$.

(i) Represent P by a digraph. (1pts)

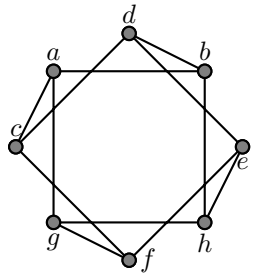
(ii) Show that P is a partial order. (3pts)

(iii) Is P a total order? (Justify your answer.) (1pts)

(iv) Represent P by a Hasse diagram. (1pts)

Q3. (a) Find the number of vertices n of a complete graph K_n which has 36 edges. (2pts)

(b) Let G be the graph below.



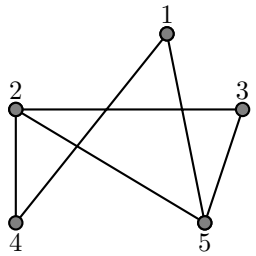
G

(i) Determine whether G is bipartite. (Justify your answer)(2pts)

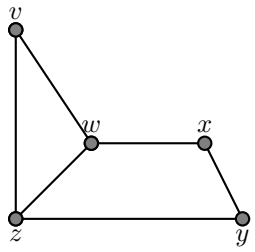
(ii) Find a simple path from a to h of length 4. (1pts)

(iii) Find a cycle from a to a of length 6. (1pts)

(c) Determine whether the following graphs M and N are isomorphic. (Justify your answer)(2pts)

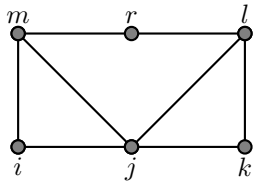


M



N

Q4. (a) For the graph H below, find a spanning tree with root r,



H

(i) using *depth-first* search; (1 pts)

(ii) using *breadth-first* search. (1 pts)

(b) Form a binary search tree for the words *english, chinese, russian, german, arabic, spanish, french.* (using alphabetical order). (2 pts)

Q5. (a) Prove the following Boolean identity. (2pts)

$$(x + y)(x + \overline{y})(\overline{x} + y) = xy.$$

(b) Find the complete sum-of-products expansion (CSP) for the function $f(x, y, z) = (x + \overline{y}z)(\overline{x} + z)$. (2pts)

(c) Find the complete product-of-sums expansion (CPS) for the function $g(x, y, z) = \overline{y} + x\overline{z}$. (2pts)

(d) Use a K-map to simplify the following sum-of-products expansion (i.e., write in MSP form).(2pts)

$$xyz + x\overline{y}\,\overline{z} + x\overline{y}z + \overline{x}y\overline{z} + \overline{x}\,\overline{y}z.$$