King Saud University
Faculty of Sciences
Department of Mathematics

Final Exam
Academic Year 2025-2026 - First Semester

Exam Information	
Course name	Real Analysis
Course Code	Math 280
Date Exam	1-1-2026
Exam Time	8: 00 PM
Duration Exam	3H
Classroom No.	1A71, Building 4
Instructor Name	Mongi Blel

Student Information	
Student's Name	
ID Number	
Serial Number	
Section Number	321
General Instructions	• Your Exam consists of 2 pages (including this page) • Keep your mobile and smart watch out of the classroom.

This section is only for instructor

#	Course Learning Outcomes (CLOs)	Related Question(s)	Points	Final Score
1	(40 points)	Q(1,2,3,4)		

Question 1 : [3+3+3=9]

1. Construct a sequence $(u_n)_n$ in $\mathbb{Q}^c$ which converges to 1.
2. Construct an increasing sequence $(v_n)_n$ in $\mathbb{Q}^c$ and a decreasing sequence $(w_n)_n$ in $\mathbb{Q}^c$ which converge to 1.

Question 2 : [3+3+3+3=12]
Study the convergence of the following series

1. $\sum_{n\geq 1} \frac{1}{n^2+\cos(n^2)}$;
2. $\sum_{n\geq 1} \frac{2n+1}{n^2+2n+3}$;
3. $\sum_{n\geq 1} \frac{1+2^n}{1+3^n}$;
4. $\sum_{n\geq 2} \frac{\sin(n)\ln(n)}{n^2}$.

Question 3 : [3+3=6]

1. Prove that the function $f(x) = 2x + \cos x$ is increasing on $\mathbb{R}$;
2. Compute $f([0,\pi])$.

Question 4 : [3]
Consider the function $f(x) = x^3 - 3x^2 + 4x - 1$.
Prove that there exists a unique $c \in [0,1]$ such that $f(c) = 0$.
(Hint: Prove that f is increasing.)

Question 5 : [3+3=6]

1. State Rolle lemma.
2. Let $f\colon [a,b] \longrightarrow \mathbb{R}$ be two times differentiable function such that $f(a) = f'(a)$ and $f(b) = f'(b)$. Consider the function g defined by: $g(x) = e^x(f(x) - f'(x))$.
   Apply Rolle's lemma to the function g to prove that there exists $c \in (a,b)$ such that $f(c) = f^{''}(c)$.

Question 6 : [3+3 =6]
Recall that $\sinh(x) = \frac{e^x - e^{-x}}{2}$ and $\cosh(x) = \frac{e^x + e^{-x}}{2}$.

1. State the Taylor formula of the functions $(\sin x - \sinh x)$ and $(\sin x.\sinh x)$ at order 3 at 0.
2. Compute $\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{\sinh x}\right)$.