

Question 1 :

$$1. F'(x) = -2x \cos \sqrt{(x^2 + 1)^2 + 1}.$$

2.

$$\begin{aligned} \int \frac{dx}{x((\ln x)^2 + 9)} &\stackrel{t=\ln x}{(1)} \int \frac{dt}{t^2 + 9} \\ &\stackrel{(1)}{=} \frac{1}{3} \tan^{-1}\left(\frac{\ln x}{3}\right) + C. \end{aligned}$$

3.

$$\int \frac{dx}{(x\sqrt{1-x^6})} \stackrel{t=x^3}{(2)} \int \frac{dt}{3t\sqrt{1-t^2}} \stackrel{(1)}{=} \frac{1}{3} \operatorname{sech}^{-1}(|x^3|) + C$$

Question 2 :

$$1. \lim_{x \rightarrow 0^+} \left(\frac{1}{x}\right)^{x^2} \stackrel{(0.5)}{=} \lim_{x \rightarrow 0^+} e^{-x^2 \ln x}.$$

$$\lim_{x \rightarrow 0^+} x^2 \ln x \stackrel{(2)}{=} 0, \quad \lim_{x \rightarrow 0^+} \left(\frac{1}{x}\right)^{x^2} \stackrel{(0.5)}{=} 1.$$

$$2. \int x^4 \ln x dx \stackrel{u=\ln x, v'=x^4}{(2+1)} \frac{1}{5} x^5 \ln x - \frac{1}{25} x^5 + c.$$

3.

$$\begin{aligned} \int (\sin x)^5 (\cos x)^6 dx &\stackrel{u=\cos x}{(2)} - \int u^6 (1-u^2)^2 du \\ &\stackrel{(1)}{=} - \int (u^6 + u^{10} - 2u^8) du \\ &= -\frac{1}{7} \cos^7 x - \frac{1}{11} \cos^{11} x + \frac{2}{9} \cos^9 x + c. \end{aligned}$$

Question 3 :

1.

$$\int \frac{dx}{(16-x^2)^{\frac{3}{2}}} \stackrel{x=4\sin t}{\underset{(2)}}{=} \frac{1}{16} \int \sec^2 t dt$$

$$\stackrel{(1)}{=} \frac{1}{16} \tan t + c = \frac{1}{16} \tan(\sin^{-1}(\frac{x}{4})) + c.$$

$$2. \int \frac{x^2+1}{x^3-x} dx \stackrel{(2)}{=} \int -\frac{dx}{x} + \frac{dx}{x-1} + \frac{dx}{x+1} \stackrel{(1)}{=} \ln \left| \frac{x^2-1}{x} \right| + c.$$

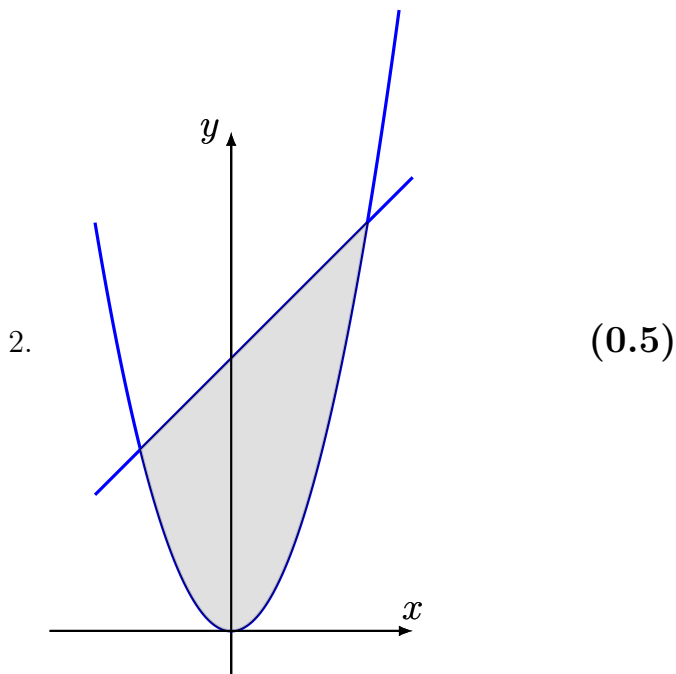
3.

$$\int \frac{1}{\sqrt{x} + \sqrt[4]{x}} dx \stackrel{x=t^4}{\underset{(1)}}{=} \int \frac{4t^2}{t+1} dt \stackrel{(2)}{=} 2t^2 - 4t + 4 \ln(t+1) + c$$

$$= 2x^{\frac{1}{2}} - 4x^{\frac{1}{4}} + 4 \ln(x^{\frac{1}{4}} + 1) + c.$$

Question 4 :

$$1. \lim_{c \rightarrow 0^+} \int_c^1 \frac{dx}{\sqrt{x}(1+x)} \stackrel{x=t^2}{\underset{(2)}}{=} \lim_{c \rightarrow 0^+} \int_{\sqrt{c}}^1 \frac{2}{1+t^2} dt = \lim_{c \rightarrow 0^+} 2 \tan^{-1}(1) - 2 \tan^{-1}(\sqrt{c}) \stackrel{(1)}{=} \frac{\pi}{2}.$$

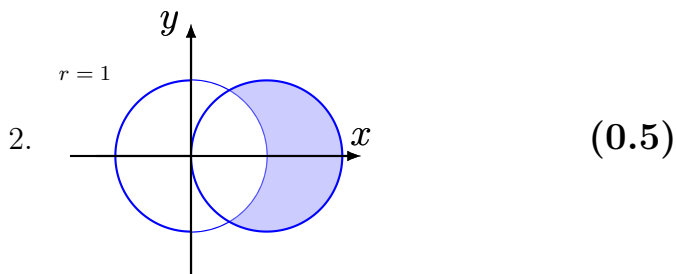


$$A \underset{(1.5)}{=} \int_{-2}^3 x + 6 - x^2 dx \underset{(1)}{=} 21 - \frac{1}{6} = \frac{125}{6}.$$

$$3. V = \pi \int_{-1}^1 (2 - x^2)^2 - x^4 dx \underset{(2)}{=} \pi \int_{-1}^1 4 - 4x^2 dx \underset{(1)}{=} \frac{16}{3} \pi.$$

Question 5 :

$$1. L = \int_0^1 \sqrt{1 + y^4 + 2y^2} dy \underset{(2)}{=} \int_0^1 (1 + y^2) dy \underset{(1)}{=} \frac{4}{3}.$$



$$A \underset{(1.5)}{=} \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (4 \cos^2(\theta) - 1) d\theta \underset{(1)}{=} \frac{\sqrt{3}}{2} + \frac{\pi}{3}.$$