Solution of the final examination Math 106, Semester I 1441-1442

Question 1 :

a)

k	x_k	$f(x_k)$	m	$mf(x_k)$
0	0	1	1	1
1	1	$\frac{1}{2}$	2	1
2	2	$\frac{1}{9}$	2	$\frac{2}{9}$
3	3	$\frac{1}{28}$	2	$\frac{1}{14}$
4	4	$\frac{1}{65}$	2	$\frac{2}{65}$
5	5	$\frac{1}{126}$	1	$\frac{1}{126}$
				2.3323

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left(f(a)+2\sum_{j=1}^{n-1} f(x_j) \quad + f(b)\right)$$

$$= \frac{5-0}{2.5}\left(1+1+\frac{2}{9}+\frac{1}{14}+\frac{2}{65}+\frac{1}{126}\right). \qquad (1.5)$$

$$\int_0^5 \frac{dx}{1+x^3} \approx \frac{1}{2}(2.3323) = 1.1661. \qquad (0.5)$$

b) $\frac{1}{6}\int_{-1}^{5}(x-2)^2dx = 3$ (1), then $c = 2 \pm \sqrt{3}$. (1)

c) For $u = \cos(x^2)$,

$$\int x\sin(x^2)3^{\cos(x^2)}dx = -\frac{1}{2}\int 3^u du \qquad (2)$$

$$= -\frac{3^{\cos(x^2)}}{2\ln 3} + c. \qquad (1)$$

Question 2 :

a)

$$\int \frac{dx}{x\sqrt{x^4-1}} \overset{t=x^2}{=} \frac{1}{2}\int \frac{dt}{t\sqrt{t^2-1}} \quad (2)$$
$$= \frac{1}{2}\sec^{-1}(x^2)+c. \quad (1)$$

b)

$$\int \frac{\cot x dx}{\sqrt{1-\sin^4 x}} \overset{t=\sin^2 x}{=} \frac{1}{2}\int \frac{dt}{t\sqrt{1-t^2}} \quad (2)$$
$$= -\frac{1}{2}\text{sech}^{-1}(\sin^2 x)+c. \quad (1)$$

c)

$$\int \cosh^{-1} x dx = x\cosh^{-1} x - \int \frac{x}{\sqrt{x^2-1}}dx \quad (2)$$
$$= x\cosh^{-1} x - \sqrt{x^2-1}+c. \quad (1)$$

Question 3 :

a)

$$\int \frac{dx}{(x^2+4)^2} \overset{x=2\tan\theta}{=} \frac{1}{8}\int \cos^2\theta d\theta$$
$$= \frac{1}{16}(\theta+\sin\theta\cos\theta)+c \quad (2)$$
$$= \frac{1}{16}\left(\tan^{-1}(\frac{x}{2})+\frac{2x}{x^2+4}\right)+c. \quad (1)$$

b)

$$\int \frac{dx}{x^{\frac{1}{2}} - x^{\frac{1}{3}}} \overset{x=t^6}{=} 6\int \frac{t^3}{t-1}dt = 6\int \left(t^2+t+1+\frac{1}{t-1}ds\right) \quad (2)$$

$$= 2\sqrt{x} + 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} + 6\ln|x^{\frac{1}{6}} - 1| + c. \quad (1)$$

c)

$$\int \frac{dx}{5 + 3\cos x + 4\sin x} \overset{t=\tan(\frac{x}{2})}{=} \int \frac{dt}{(t+2)^2} \quad (2)$$

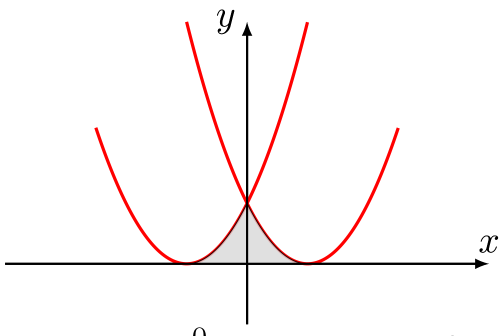
$$= -\frac{1}{\tan(\frac{x}{2}) + 2} + c. \quad (1)$$

Question 4 :

a) $\int_0^a (1+x)e^{-2x}dx = \frac{3}{4} - \frac{3}{4}e^{-2a} - \frac{a}{2}e^{-2a}$ (2).

So the integral converges and $\int_0^{+\infty} (1+x)e^{-2x}dx = \frac{3}{4}$. (1)

b) Graph: (1)



$$A = 2\int_{-1}^{0} (x+1)^2 dx = \frac{2}{3}. \quad (2)$$

Question 5 :

The volume obtained by revolving the region bounded by $y = x^2$ and $y = \sqrt{x}$ about the $x-$axis is

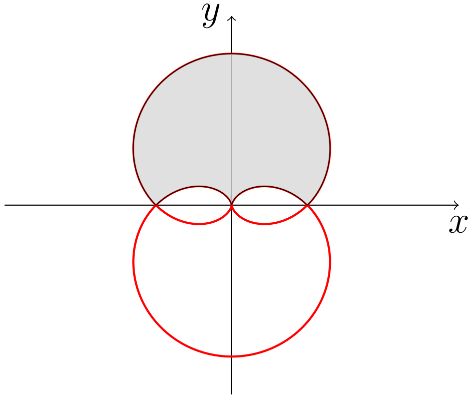
$$V = \pi \int_0^1 (x - x^4) dx = \pi \left(\frac{1}{2} - \frac{1}{5} \right) = \frac{3\pi}{10}.$$

(Intersection:(0.5)
Integral: (2)
result (0.5)).

Question 6 :
a)

$$SA = 2\pi \int_0^1 4t\sqrt{16t^2 + 16} dt = 32\pi \int_0^1 t\sqrt{t^2+1} dt. \quad (1.5)$$
$$= 16\pi \int_1^2 \sqrt{u} du \quad (1)$$
$$= \frac{32\pi}{3} \pi (2\sqrt{2} - 1) \quad (0.5)$$

b) Graph: (1)



$$\begin{aligned} A &= \frac{1}{2}\int_0^{\pi} (1+\sin\theta)^2 - (1-\sin\theta)^2 d\theta \qquad \color{red}{(1)} \\ &= 2\int_0^{\pi} \sin\theta d\theta = 4. \qquad \color{red}{(1)} \end{aligned}$$