King Saud University	College of Sciences	Department of Mathematics	
Final Examination	Math 228	Semester 2 (1443)	Time: 3 Hours

The Examination contains 2 pages

Question 1: (3+3+3)

1. Find the area A of the region in the xy-plane bounded by the graphs $y = 4x - x^2$ and $y = -x$.
2. Evaluate the integral $\int_0^2 \int_0^y y^4 \cos(xy^2)\ dx\ dy$.
3. Evaluate the integral $\iint\limits_D e^{x^2+y^2}\ dA$, where D is the disc of center $(0,0)$ and radius 2.

Question 2: (3+3+3+3)

1. Find the volume of the tetrahedron bounded by the coordinates $x = 0$, $y = 0$, $z = 0$ and the plane $x + y + z = 1$.
2. Evaluate the integral by changing to spherical coordinates
$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2 + y^2 + z^2}\ dz\ dx\ dy$$
3. Use cylindrical coordinates to evaluate $\iiint\limits_Q \sqrt{x^2 + y^2} dV$, where Q is the solid bounded by the paraboloid $z = 1 - (x^2 + y^2)$ and the xy-plane.
4. Find the volume of the region that lies between two spheres $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 = 4$.

Question 3: (3+2+2+3+3)

1. Find the sum of the series $\sum_{n=1}^{\infty} \left[\frac{1}{(n+3)(n+4)} + \frac{1}{2^n}\right]$.
2. Determine whether the series $\sum_{n=1}^{\infty} \frac{100^n}{n!}$ converges or diverges. Justify you answer.
3. Use integral test to determine whether the series $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$ converges or diverges.
4. Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2 + 4}$ is absolutely convergent, conditionally convergent, or divergent.

5. Find the interval of convergence and radius of convergence of the power series $\sum_{n=1}^{\infty}(-1)^n \frac{x^n}{n4^n}$.

Question 4: (3+3)

1. Find the power series representation of $f(x) = \ln(1+x)$, $|x| < 1$, and use first three non zeros terms to find the value of $\ln(1.2)$.
2. Gives the Maclaurin's series for the function $f(x) = \cos x$ and prove that it represents $\cos x$ for all x. hence approximate the integral $\int_0^1 x\cos(x^3)\,dx$ up to four decimal places by using the first three non-zeros terms.