The final examination **King Saud university**

First semester, 1431H **Math 244**

Time: 3 hours

....

Question No.1

(a) Determine whether the following statements are true or false, and justify your answer:

(1) The set of vectors $\{v_1 = (-2,0,1), v_2 = (3,2,5), v_3 = (6,-1,1), v_4 = (7,0,-2)\}$ is a basis of $\mathbb{R}^3$.

(2) If $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ is the orthogonal projection on the X-axis and $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ is the orthogonal projection on the Y-axis, then $T_1 \circ T_2 = T_2 \circ T_1$.

(3) Whenever 2 and 4 are eigenvalues of a matrix A, then the eigenvalues of A^3 are 6 and 12.

(4) If $Ax = b$, where $A = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$, then x equals $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$.

(5) If B is 5×7 matrix and $\text{Rank}(B) = 3$, then $\text{rank}(B^T)$ equals to 3.

(6) If $u = (-2,3,1,4)$ and $v = (1,2,0,-1)$ two vectors in $\mathbb{R}^4$, then u and v are orthogonal.

(b) Choose the correct answer:

(1) For any two invertible matrices A and B, $(AB^{-1})^{-1}$ is equal to

$(i) A^{-1}B$ $(ii) B^{-1}A$ $(iii) AB^{-1}$ $(v) BA^{-1}$

(2) If $A = \begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}$, then A^{-2} is equal to

$(i) \begin{pmatrix} -3 & 2 \\ 1 & -\frac{1}{2} \end{pmatrix}$ $(ii) \begin{pmatrix} 11 & -7 \\ -\frac{7}{2} & \frac{9}{4} \end{pmatrix}$ $(iii) \begin{pmatrix} 6 & -4 \\ -2 & 1 \end{pmatrix}$

(3) If $u = (2, \frac{-3}{2}, 0, \frac{1}{2}, -\frac{1}{2}, 3)$, then $\| -2u\|$ equals:

$(i)5$ $(ii)\dfrac{3}{2}$ $(iii)1$ (v) None of these

(4) The standard matrix of the orthogonal projection on the YZ- plane is

$(i) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $(ii) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $(iii) \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(5) If $A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix}$, then the distinct eigenvalues of A are

$(i)2$ and 4 $(ii)0$ and 2 $(iii)0$ and 4 (v) None of these .

....

Question No.2

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x - 2y + 2z, 2x + y + z, x + y)$.

(a) Find the standard matrix of T.

(b) Show that T is one-to-one.

(c) Find $T^{-1}(w_1, w_2, w_3)$.

....

Question No.3

(a) Find the eigenvalues of A^5 for

$$A = \begin{pmatrix} 1 & 3 & 7 & 11 \\ 0 & \frac{1}{2} & 3 & 8 \\ 0 & O & 0 & 4 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

Is A^5 invertible ? why ?

(b) If $A = \begin{pmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{pmatrix}$, then

(1) Find the rank and nullity of A.

(2) Find a basis of the null space of A.

....

Question No.4

(a) Show that the following set of vectors is a basis for M_{22}:

$$\{\begin{pmatrix} 3 & 6 \\ 3 & -6 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -8 \\ -12 & -4 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}\}.$$

(b) Solve the following linear system by Gauss-Jordan elimination:

$$x-y+2z-w=-1, 2x+y-2z-2w=-2, -x+2y-4z+w=1, 3x-3w=-3.$$

....

Good luck