King Saud University

Faculty of Sciences

Department of Mathematics

Final Examination    Math 106    Semester I - 1443
Time: 3H

## Question 1 : (2+2)

a) If $F(x) = \int_{x^2}^{\sin x} \cosh t\, dt$, find $F'(0)$.

b) Find the number(s) $z$ in the mean value theorem for $f(x) = \dfrac{1}{x^2}$ on $[1, 2]$.

## Question 2 : (3+3+3)

a) Evaluate $\displaystyle\int \frac{dx}{\sqrt{3^{2x} - 1}}$.

b) Compute $\displaystyle\int \frac{\cot x}{\sqrt{1 - \sin^2 x}}\,dx$.

c) Find $\displaystyle\lim_{x \to 0^+} x^{\sin x}$.

## Question 3 : (3+3+3)

a) Compute the integral $\displaystyle\int x \tan^{-1} x\, dx$.

b) Evaluate $\displaystyle\int \frac{dx}{(1 + x^2)^2}$.

c) Find $\displaystyle\int \frac{dx}{\sqrt{x^2 + 8x + 25}}$.

**Question 4 : (3+3+3)**

a) Evaluate the integral $\int \dfrac{dx}{x^{\frac{1}{4}} + x^{\frac{1}{2}}}$.

b) Does the integral $\int_{2}^{+\infty} \dfrac{dx}{x(\ln x)^3}$ converge? Find its value if it does.

c) Sketch the region bounded by the curves $y = x^2 + 1$, $y = -x^2 + 1$, $x = 2$ and find its area.

**Question 5 : (3+3+3)**

a) Find the volume obtained by revolving the region in the first quadrant bounded by $y = 4 - x^2$, $y = 0$ and $x = 0$ about the $y$-axis.

b) Find the surface area obtained by revolving the parametric curve $x = t^3$, $y = 2t + 3$, $0 \le t \le 1$, about the $y$-axis.

c) Sketch the region inside $r = 1 + \cos\theta$ and outside $r = 1 - \cos\theta$ and find its area.

**Final exam Math106**

## Question 1(2+3)

a) Let $F(x) = \int_2^{e^x} \ln t \, dt$. Find $F'(1)$.

b) Find the number(s) $c$ that satisfies the conclusion of the mean value theorem
for the function $f(x) = \dfrac{x}{\sqrt{x^2+16}}$ on $[0, 3]$.

## Question 2(2+3+3)

a) Evaluate the integral $\displaystyle\int \dfrac{5^x \, dx}{5^{2x}+4}$

b) Compute the integral $\displaystyle\int \dfrac{x-3}{x\sqrt{x^2-25}} \, dx$

c) Find the indefinite integral $\displaystyle\int \sinh^{-1} x \, dx$

## Question 3(3+3+3)

a) Find $\lim_{x \to +\infty} (e^x + x)^{\frac{1}{x}}$

b) Evaluate the integral $\displaystyle\int \dfrac{\sqrt{9-x^2}}{x^2} \, dx$

c) Compute $\displaystyle\int \dfrac{9x^2-20x+10}{x^3-3x^2+2x} \, dx$

## Question 4(3+3+3)

a) Find $\int \frac{dx}{\sqrt{x^2+2x+5}}$

b) Does the integral $\int_1^3 \frac{dx}{\sqrt[3]{x-1}} dx$ converge? Find its value if it does.

c) Sketch the region bounded by the curves $x = y^2$ and $x = 2 - y^2$ and find its area.

## Question5(3+3+3)

a) Find the volume of the solid obtained by revolving the region bounded by $y = 4 - x^2$ and $y = 4 - 2x$ about the y-axis.

b) Compute the arc length of the curve $y = \cosh x$, $0 \le x \le 1$.

c) Sketch the region inside $r = 2\sin\theta$ and outside $r = 2 - 2\sin\theta$ and find its area.