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|  | **5411 Math** | **King Saud University** |
| **27/4 /2020** | **Final Exam** | **Second Term** |

**Q1:** Prove the followings:

1. Let $R$ be a ring with identity. If $B$ is a unitary left $R-$module then $R ⨂\_{R}B≅ B $as left $R-$modules.
2. If $A → B → C ⟶\left\{0\right\} $ is an exact sequence of right modules over a ring $R$ and $D$ is a left $R-$ module then: $A ⨂\_{R}D→B⨂\_{R}D→ C⨂\_{R}D ⟶ \left\{0\right\} $is an exact sequence of abelian groups.

**Solve 2 of the following questions only:**

**Q2:** Prove the followings:

1. Every free $R$- modules $F $is projective.
2. ) P is a projective module *iff* every short exact sequence of $R-$ modules

$$0\rightarrow  A→B→P \rightarrow 0 is splits.$$

**Q3:** Prove the followings:

1. ) Q is injective *iff* Any short exact sequence $0\rightarrow Q⟶M⟶M^{'} ⟶0 $ of $R-$modules splits.
2. The $Z-$module${ Q}/{Z}$ is injective (prove any theorem you used).

**Q4:**

1. Prove that if $0→N→M→L→0$  is an exact sequence of $R-$modules. Then

$M$ is Noetherian $⇔$ $N$ and $L$ are Noetherian.

1. Show that if $A$ is a commutative ring with identity, and $A$ is an Artinian ring then every prime ideal is maximal.

***Good luck!***