

Final Exam - Allowed time: 3 hours  
Calculators are not permitted

Q1.

[3] (a) Let  $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$ . Compute, if possible,  $AB$  and  $BA$ .

[2] (b) Compute the determinant  $\begin{vmatrix} 1 & 1 & -1 \\ 0 & -2 & -5 \\ 1 & 2 & 1 \end{vmatrix}$ .

[4] (c) Solve by using Gauss-Jordan Elimination Method the linear system

$$\begin{cases} x + y - z = -3 \\ -2y + 5z = 1 \\ x + 2y + z = 1 \end{cases}$$

Q2.

[4] (a) Find the standard equation of the ellipse with endpoints of minor axis are  $(1, 4)$  and  $(1, -2)$  and the distance between foci is 8, and then sketch its graph.

[3] (b) Find the elements of the conic section  $y = 4x - x^2$  and then sketch it.

Q3.

[2,3,3] (a) Compute the integrals:

(i)  $\int 8x(x^2 + 24)^3 dx$ , (ii)  $\int (\ln x)^2 dx$ , (iii)  $\int \frac{3x}{x^2 - 2x - 8} dx$ .

[3] (b) Sketch the region bounded by the curves  $y = x^2$ ,  $y = 2x + 3$ ,  $x = 1$  and  $x = 2$  and compute its area.

[4] (c) The region bounded by the curves  $y = 4x - x^2$  and  $y = x$  is rotated about the  $y$ -axis to form a solid  $S$ . Use the method of cylindrical shells to find the volume of  $S$ .

[2] (d) Give the Cartesian coordinates of the points in polar coordinates

$$M\left(\sqrt{2}, \frac{\pi}{4}\right) \quad \text{and} \quad N(2, \pi).$$

Q4.

[3] (a) Let  $z = xy^2 + \sin(xy)$ , where  $x = s^2t$  and  $y = \frac{t}{s}$ . Use the chaine rule to compute the partial derivatives  $\frac{\partial z}{\partial s}$  and  $\frac{\partial z}{\partial t}$ .

[4] (b) Solve the differential equation:  $xy' + y = 3x^2 + 1$ .