King Saud University
Department of Mathematics

Final Exam in Math151, Semester 1, 1444H.
Calculators are not allowed
(The exam is two-pages long)

Q1. **(a)** Without using truth tables, show that $(p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee q)$ is logically equivalent to $p \wedge q$. (3pts)

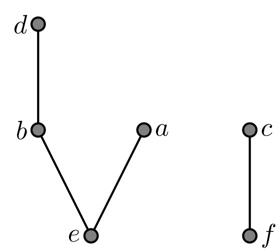
(b) Use induction to show the following for every $n \geq 1$:

$$2 + 8 + 14 + \cdots + (6n - 4) = n(3n - 1). \text{ (4pts)}$$

(c) Prove by contradiction: "For $m \in \mathbb{Z}$, if $m^2 - 3m + 5$ is even, then m is even". (2pts)

Q2. **(a)** Let R be the relation on $\mathbb{Z}$ defined by mRn if and only if 10 divides $m^4 - n^4$.
(i) Show that R is an equivalence relation. (3pts)
(ii) Show that $(m, -m) \in R$ for every $m \in \mathbb{Z}$. (1pts)
(iii) Is $[3] = [-1]$? (Justify your answer.) (1pts)

(b) Let P be the partial order on $A = \{a, b, c, d, e, f\}$ represented by the following Hasse diagram.



(i) List all ordered pairs of P. (2pts)
(ii) Is P a total order? (Justify your answer.) (1pts)

(c) Let $S = \{(1,1), (1,2), (2,3), (3,2), (3,4), (4,4), (5,5)\}$ be a relation on $B = \{1, 2, 3, 4, 5\}$.
(i) Represent S by a digraph. (1pts)
(ii) Is S antisymmetric? (Justify your answer.) (1pts)

Q3. **(a)** Let $G = (V, E)$ be an r-regular graph with $|V| = |E|$. Find the value of r. (1pts)

(b) Let H be a graph with degree-sequence $b-1$, b, $b+1$, $b+2$. Find the value of b if H has 9 edges. (2pts)

(c) Let N be the simple graph represented by the following adjacency matrix.

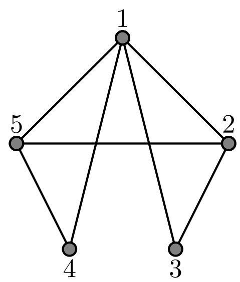
$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(i) Draw N. (1pts)

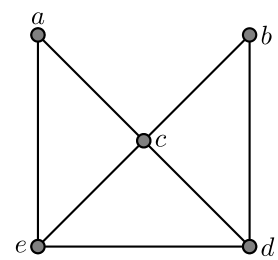
(ii) Is N connected? (Justify your answer.) (1pts)
(iii) Determine whether N is bipartite. If so, give a bipartite representation. (2pts)

(d) Determine whether the following graphs I and J are isomorphic. (2pts)

I

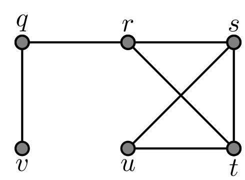


J



Q4. (a) For the graph L below, find a spanning tree with root r,
(i) using *depth-first* search; (1pts)
(ii) using *breadth-first* search. (1pts)

L



(b) (i) Using alphabetical order, form a binary search tree for the words *Makkah, Madinah, Riyadh, Jeddah, Qassim, Al-Khobar, Dammam.* (2 pts)

(ii) Is the tree in (i) full binary? (Justify your answer.) (1pts)

Q5. (a) Let $f(x, y, z) = (\overline{x} + \overline{y})(x + z)$ be a Boolean function.
(i) Find the complete sum-of-products expansion (CSP) of f. (2pts)
(ii) Find the complete product-of-sums expansion (CPS) of f. (2pts)

(b) Let $g(x, y, z) = xyz + xy\overline{z} + x\overline{y}z + \overline{x}y\overline{z} + \overline{x}\,\overline{y}\,\overline{z}$ be a Boolean function.
(i) Build the K-map of g. (1pts)
(ii) Simplify g (i.e., write in MSP form). (2pts)