

King Saud University, Department of Mathematics
Math 204 (3H), 40/100, Final Exam

Question 1[4,4] a) Obtain the solution of the differential equation

$$e^x y y' = e^{-y} + e^{-2x-y}.$$

b) Use the substitution $u = \ln y$, to solve the differential equation

$$x y' = 2x^2 y + y \ln y, \quad y > 0.$$

Question 2[4,4] a) Find the family of orthogonal trajectories of the family of curves

$$\sqrt{y - C} = 1 - \sqrt{x}.$$

b) Use the undetermined coefficients method to solve the differential equation

$$y'' - 4y' + 4y = 3xe^{2x}.$$

Question 3[4,5,5] a) If $y_1 = x$ is a solution of the differential equation

$$x^2(1 - \ln x)y'' + xy' - y = 0, \quad \text{for } x > e,$$

then find its general solution.

b) Compute the first five coefficients of the power series solution about the origin for the problem

$$\begin{cases} y'' + y' + x^2 y = 0, \\ y(0) = 1, \quad y'(0) = 2 \end{cases}$$

c) Obtain the general solution of the differential equation

$$x^2 y'' - 2y = x^2 + \frac{1}{x}, \quad x > 0.$$

Question 4[5,5] a) Find the Fourier series for the periodic function of period 4

$$f(x) = 2 - |x|, \quad x \in [-2, 2].$$

Deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

b) Sketch the graph of the following function and find its Fourier integral

$$f(x) = \begin{cases} 0, & x < -1 \\ x + 1, & -1 \leq x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ 0, & x > 1 \end{cases} \quad \text{Deduce that } \int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \int_0^{\infty} \frac{\sin x}{x} dx.$$