KING SAUD UNIVERSITY DEPARTMENT OF MATHEMATICS
M204.TIME 3H, FULL MARKS 40, FINAL EXAM S2-2024/25

Question 1. **[5,4,4] a)** Solve the initial value problem

$$\begin{cases} y' = 2xy^2 + 3y^2 - 8x - 12 \\ \qquad\qquad y(0) = -1 \end{cases}$$

b) Solve the differential equation

$$y \cos x + 2xe^y + (\sin x + x^2 e^y - 1)y' = 0.$$

c) A metal sphere, when suspended in a constant temperature enclosure cools from 80^0C to 70^0C in 5 minutes and to 62^0C in the next 5 minutes. Calculate the temperature of the enclosure.

Question 2. **[4,4] a)** Obtain the general solution of the differential equation

$$y''' + 4y' = x + e^{2x} + 3\sin x.$$

b) Find the general solution of the differential equation

$$y'' - 2y' + y = \frac{e^x}{1 + x^2}.$$

Question 3. **[4,5] a)** Solve the system of differential equations

$$\begin{cases} y'' + x' + x = 0 \\ y' - y + x = t^2. \end{cases}$$

b) Find the first five nonzero terms in a power series expansion about $x_0 = 0$ for the solution to the given initial value problem

$$y'' - xy' + 2y = 0, \quad y(0) = 1, \ y'(0) = 1.$$

Question 4. **[5,5] a)** Consider the function g defined by

$$g(x) = \begin{cases} 0 & \text{if } -1 < x < 0 \\ 1 - x & \text{if } \quad 0 < x < 1 \end{cases}.$$

such that $g(x + 2) = g(x)$. Find the Fourier series of g, and deduce the value of the numerical series

$$\sum_{n=0}^{\infty} \frac{1}{(2n + 1)^2}.$$

b) Find the Fourier sine integral of the function

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \qquad x > 1 \end{cases}$$

and deduce the value of the integral $\displaystyle\int_0^{\infty} \frac{\sin^3 x}{x} dx..$

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