KING SAUD UNIVERSITY,DEPARTMENT OF MATHEMATICS
MATH 204. TIME: 3H, FULL MARKS: 40, FINAL EXAM

Question 1. [5,4] a) Initial population of a town increases by 1% in first two years and becomes 10000 in four years. What is the initial population, if the rate of growth of the population is directly proportional to the population at that instant?.

b) Find the general solution of the differential equation

$$xy^4dx + (2+y^2)e^{-3x}dy = 0,\ y > 0.$$

Question 2. [4,5] a) For the differential equation

$$(-xy\sin x + 2y\cos x)dx + 2x\cos x dy = 0,\ x \neq 0,\ y \neq 0.$$

verify that $\mu(x,y) = xy$ is an integrating factor, hence solve it.

b) Use power series method to find the first four terms of the solution for the initial value problem

$$(x+1)y'' = 1,\quad y(0) = 0,\ y'(0) = 1.$$

Question 3. [5,5] a) Consider the $2\pi-$periodic function $f(x) = x$, for $x \in (-\pi,\pi]$.

Sketch the graph of f on $(-3\pi, 3\pi)$, obtain the Fourier series for the function f, and deduce the value of the numerical series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \qquad (\text{Hint} : \sin\frac{n\pi}{2} \begin{cases} 0, & n = 2k \\ (-1)^k, & n = 2k+1 \end{cases}$$

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b) Consider the function: $f(x) = \begin{cases} \cos x, & |x| < \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases}$

Sketch the graph of f, find the Fourier integral representation, and deduce tha $\int\limits_0^{\infty} \frac{\cos(\frac{\pi\lambda}{2})}{1-\lambda^2} d\lambda = \frac{\pi}{2}$.

Question 4. [5,3,4] a) Find the largest interval for which the following initial value problem admits a unique solution

$$\begin{cases} \frac{x^2}{x^2+4}y'' + \frac{x+3}{(4-x)^{1l3}}y' + \frac{2y}{\sqrt{x-2}} = 0 \\ y(5) = 1,\ y'(5) = -2 \end{cases}$$

b) Determine the geneal solution of the homogeneous differential equation having the characteristic equation

$$(m^4-1)(m-1)^4m^4 = 0.$$

c) Solve the the differential equation $y'' + 2y' + y = e^{-x}\ln x,\ x > 0.$