King Saud university **Second semester, 1431H**

Time: 3 hours **Math 202**

The final examination

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Question No.1

Determine whether the following statements are true or false, and justify your answer:

(a) The line integral $\int_C 5ydx + 5xdy + yz^2dz$ is independent of path.

(b) $\vec{i} + \vec{j} + \vec{k}$ is a unit vector.

(c) If $r = x\vec{i} + y\vec{j} + z\vec{k}$, then $\nabla \times r = 3$, and $\nabla.r = 0$

(d) $(\vec{k} \times \vec{i}) \times \vec{i} = \vec{k}$

(e) the symmetric form for the line through the points $P_1(3, 1, -2)$ and $P_2(-2, 7, -4)$:

$$\frac{x-3}{-2} = \frac{y-1}{7} = \frac{z+2}{-4}$$

(f) The graph of $\frac{-x^2}{4} + \frac{y^2}{9} = 1$ is hyperbola.

Find the polar equation for $(x^2 + y^2)^{\frac{3}{2}} - 4xy = 0$, and then sketch the graph.

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Question No.2

(a) Find the surface area of the paraboloid given by

$$z = 4 - x^2 - y^2, \quad \text{for } z \geq 0$$

(b) Evaluate the line integrals $\int_C xy^{\frac{2}{5}}dS$, if C has the parametrization $x = \frac{1}{2}t$, $y = t^{\frac{5}{2}}$, $0 \leq t \leq 1$.

(c) Show that $\int_C F.dr$ is independent of path by finding a potential function f for F, where

$$F(x,y,z) = y^2\cos x\vec{i} + (2y\sin x + e^{2z})\vec{j} + 2ye^{2z}\vec{k}$$

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Question No.3

(a) Let f be a scalar function and F a vector function. If the partial derivatives exist, Show that

$$\nabla.(fF) = f(\nabla.F) + (\nabla f).F$$

(b) Find the curvature of the curve $y = \ln(x-1)$ at $P(2,0)$.

(c) If r is differentiable and $\|r(t)\|$ is constant, prove that $r'(t)$ is orthogonal to $r(t)$, for all $t \in Dom(r')$.

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Question No.4

1. Use Green's theorem to evaluate $\oint_C 5xydx + x^3dy$, where C is the closed curve consisting of the graphs of $y = x^2$ and $y = 2x$ between the points $(0,0)$ and $(2,4)$.

2. Let S be the part of the paraboloid $z = 9 - x^2 - y^2$ with $z \geq 0$, and let C be the trace of S on the XY-plane. Verify Stokes's theorem for the vector field

$$F = 3z\vec{i} + 4x\vec{j} + 2y\vec{k}$$

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Good luck